International Journal of Qualitative Theory of Differential Equations and Applications
Vol. 3, No. 1-2 (2009), pp. 3–7

Creating a chaos in a system with relay

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Received October 10, 2008; accepted April 21, 2009

Abstract. We address a special initial value problem of a differential equation with relay function. The concept of Li-Yorke chaos [8] is considered.

AMS Subject Classifications: 34C28, 37D45, 34A37, 34C25, 34D20

Keywords: Quasilinear systems; Systems with relay; Li-Yorke chaos.

1. Introduction and Preliminaries

The method of construction of chaotic motions has been proposed in [1]-[3]. We consider a special initial value problem for relay systems and impulsive systems, whose initial moments of time are from a Cantor set. Using the map, which is topologically conjugate to symbolic dynamics, as the generator of moments of the relay switching in the multidimensional system, we observe in paper [1] Devaney's ingredients of chaos for a relay system with linear elements. Existence of a quasi-minimal set has been proved in [3]. The approach has been used, also, in [2] to construct the Li-Yorke chaos [8] for impulsive differential equations. In the present article we attempt to shape the chaos for the multidimensional non-linear relay system.

Let us recall the definition of chaos for maps. Consider an infinite nonvoid compact metric space (X, ρ) with metric ρ and a continuous map $T : X \to X$. A pair $(x, x') \in X \times X, x \neq x'$, is called a *Li-Yorke pair* [5] if it is *proximal* and *not asymptotic*, that is, $\liminf_{i\to\infty}\rho(T^i(x), T^i(\tilde{x})) = 0$ and $\limsup_{i\to\infty}\rho(T^i(x), T^i(\tilde{x})) > 0$, respectively.

The map $T : X \to X$ is Li-Yorke chaotic, if: it has points with all periods $p \in \mathbb{N}$; there exists an uncountable subset $X' \subset X$, the scrambled set, that does not contain periodic points and each pair $(x, x') \in X' \times X', x \neq x'$, is a Li-Yorke pair. Consider the sequence space [9]

$$\Sigma_2 = \{s = (s_0 s_1 s_2 \ldots) : s_j = 0 \text{ or } 1\}$$

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