PERIODIC SOLUTIONS OF QUASILINEAR IMPULSE SYSTEMS IN THE CRITICAL CASE

M. U. Akhmetov and N. A. Perestyuk

UDC 517.911

Conditions for the existence of periodic solutions of weakly-nonlinear autonomous and nonautonomous impulse systems in the critical case are determined.

To construct periodic motions in systems subjected to impulse action one often resorts to the procedure in which instantaneous changes take place at instants when the parameters of these systems satisfy some previously established relations [1, 2]. Then the instants of impulse action are determined not only by properties of the constraint that is external with respect to the system under study, but also by internal constraints. Mathematical models for such systems are differential equations with impulse action at nonfixed instants. The basic results of the theory of differential equations with impulse action at nonfixed instants are treated in [3].

In this paper we generalize to equations of this kind the theorems of existence of periodic solutions in the critical case for differential equations [4].

1. Periodic Solutions of Nonautonomous Systems. Consider a system of differential equations with impulses of the form

$$\frac{dx}{dt} = A(t)x + f(t) + \mu F(t, x, \mu), \quad t \neq t_i + \mu\tau_i(x, \mu),$$
$$\Delta x|_{t=t_i+\mu\tau_i(x,\mu)} = B_i x + I_i + \mu W_i(x, \mu), \tag{1}$$

where $x \in R^n$, μ is a small parameter, $A(t)$, $t \in R$, and B_i, $i \in Z$ (the set of integers) are $n \times n$ matrices, $F \in C^{(0,1,2)}$, W_i, $\tau_i \in C^{(1,2)}$. There exist a real number $\omega > 0$ and an integer p such that for any x, μ, t, i one has the relations

$$A(t+\omega) = A(t), \quad f(t+\omega) = f(t), \quad F(t+\omega, x, \mu) = F(t, x, \mu),$$
$$t_{i+p} = t_i + \omega, \quad B_{i+p} = B_i, \quad \tau_{i+p}(x, \mu) = \tau_i(x, \mu), \quad I_{i+p} = I_i,$$
$$W_{i+p}(x, \mu) = W_i(x, \mu).$$

Before we continue the investigation of system (1), let us formulate a number of definitions used below [5-8].

Let $x_j(t)$, j = 1, 2 be solutions of system (1), and let t_i^j be the discontinuity points of these solutions. We say that the solution $x_1(t)$ lies in the ε-neighborhood of the solution $x_2(t)$ if: 1) the measure of the symmetric difference of the domains of existence of the two solutions is no larger than ε; 2) the inequality $|t_i^1 - t_i^2| < \varepsilon$ holds for all i; 3) the inequality $\|x_1(t) - x_2(t)\| < \varepsilon$ holds for all t such that $|t - t_i^2| > \varepsilon$.

The topology defined by means of these ε-neighborhoods will be referred to as the B-topology. This topology is Hausdorff and can be defined also in the case where the solutions exist on the real half-line or line.

We say that a piecewise continuous function $u_j(t)$ is a B-derivative of the solution $x(t) = x(t, t_0, x_0)$ of system (1) with respect to x_0^j, $j = \overline{1, n}$, $x_0 = (x_0^1, x_0^2, \ldots, x_0^n)$, if the function $\xi u_j(t)$ lies in the θ-neighborhood of the difference $x_j(t) - x(t)$, where $x_j(t)$ is the solution of system (1) with initial condition $x_j(t_0) = (x_0^1, \ldots, x_0^j + \xi, \ldots, x_0^n)$ and $\theta \to 0$ as $\xi \to 0$. Furthermore, for all t lying outside the θ-neighborhood of the discontinuity points of x(t) one has the inequality $\|x_j(t) - x(t) - \xi u_j(t)\| < \theta_1$, where $\theta_1 = o(\xi)$.

Translated from Ukrainskii Matematicheskii Zhurnal, Vol. 43, No. 3, pp. 308-315, March, 1991. Original article submitted November 9, 1989.

By analogy with the definition of the B-derivative with respect to the initial condition one can give the definition of the B-derivatives with respect to the initial time t_0 and the parameter μ. Solutions of impulse systems are here assumed to be left-continuous at every point in the domain of existence.

Now let φ_i, $i = \overline{1, k}$, $k < n$, be solutions of the linear homogeneous system corresponding to system (1),

$$\frac{dx}{dt} = A(t)x, \quad t \neq t_i, \quad \Delta x|_{t=t_i} = B_i x, \tag{2}$$

which form a maximal set of linearly independent ω-periodic solutions. Then the system transposed to (2),

$$\frac{dy}{dt} = -A^T(t)y, \quad t \neq t_i, \quad \Delta y|_{t=t_i} = -(E + B_i^T)^{-1} B_i^T y \tag{3}$$

(where T denotes transposition), also has k linearly independent ω-periodic solutions ψ_j, $j = 1, \ldots, k$ [3].

From these solutions we compose an $n \times k$ matrix $H_1(t)$. Suppose that there is satisfied the condition

$$\int_0^\omega H_1^T(t) f(t)\, dt + \sum_{i=1}^p H_1^T(t_i) I_i = 0.$$

Then according to [3] the equation

$$\frac{dx}{dt} = A(t)x + f(t), \quad t \neq t_i, \quad \Delta x|_{t=t_i} = B_i x + I_i \tag{4}$$

has a family of ω-periodic solutions $x(t, \alpha) = \alpha_1\varphi_1 + \alpha_2\varphi_2 + \ldots + \alpha_k\varphi_k + \varphi_0$, where φ_0 is a particular ω-periodic solution of Eq. (4).

THEOREM 1. Suppose system (1) satisfies the conditions discussed and Eq. (4) admits a family of ω-periodic solutions $x(t, \alpha)$.

Let α_0 be a solution of the equation $h(\alpha) = 0$, where

$$h(\alpha) = \int_0^\omega H_1^T(t) F(t, x(t, \alpha), 0)\, dt +$$

$$+ \sum_{i=1}^p H_1^T(t_i)\{W_i(x(t_i, \alpha), 0) + (E + B_i)\tau_i(x(t_i, \alpha), 0) A(t_i)[B_i x(t_i, \alpha) + I_i]\}$$

such that

$$\det\left[\frac{\partial h}{\partial \alpha}\bigg|_{\alpha=\alpha_0}\right] \neq 0.$$

Then for sufficiently small $|\mu|$ Eq. (1) has an ω-periodic solution that converges in the B-topology to $x(t, \alpha_0)$ when $\mu \to 0$.

Proof. Let $x_1(t) = x(t, t_i, x, \mu)$ be the solution of Eq. (1) and let τ_i be the time at which this solution meets the surface $t = t_i + \mu\tau_i(x, \mu)$. Let $x_2(t) = x(t, \tau_i, x_1(\tau_i+), \mu)$ be a solution of the same equation, defined on the segment $[\tau_i, t_i]$ if $\tau_i \leq t_i$, or $[t_i, \tau_i]$ if $t_i < \tau_i$. We shall assume that the solution $x_2(t)$ is not subjected to impulse action. Denote

$$\mu\Omega_i(x, \mu) = (E + B_i)\int_{t_i}^{\tau_i} (A(u)x_1(u) + f(u) + \mu F(u, x_1(u), \mu))\, du +$$

$$+\mu W_i\left(x+\int_{t_i}^{\tau_i}(A(u)x_1(u)+f(u)+\mu F(u,x_1(u),\mu))\,du\right)+$$

$$+I_i+\int_{\tau_i}^{t_i}(A(u)x_2(u)+f(u)+\mu F(u,x_2(u),\mu))\,du.$$

One can verify that $\Omega_i \in C^{(1,1)}$ and

$$\Omega_i(x,0)=W_i(x,0)+(E+B_i)\tau_i(x,0)A(t_i)[B_ix+I_i]. \tag{5}$$

Moreover, for any solution of system (1) with discontinuities at the points η_i, $i = 1, \ldots, p$, $0 < \eta_1 < \eta_2 < \ldots < \eta_p < \omega$ there is a solution $\xi(t)$ of the equation

$$\frac{dx}{dt}=A(t)x+f(t)+\mu F(t,x,\mu),\quad t\neq t_i,$$
$$\Delta x|_{t=t_i}=B_ix+I_i+\mu\Omega_i(x,\mu), \tag{6}$$

such that $x(\eta_i) = \xi(\eta_i)$ or $x(\eta_i+) = \xi(\eta_i)$ for all $i = 1, \ldots, p$.

Now let us investigate the question of the existence of a periodic solution of system (6).

Complete the matrix $H_1(t)$ by columns that are solutions ψ_j, $j = k + 1, \ldots, n$, of system (3) to obtain a fundamental matrix of solutions $H(t)$. Performing the substitution $y = H^T(0)x$ in Eqs. (6) we obtain the system

$$\frac{dy}{dt}=P(t)y+g(t)+\mu\mathcal{F}(t,y,\mu),\quad t=t_i,$$
$$\Delta y|_{t=t_i}=Q_iy+J_i+\mu\mathcal{P}_i(y,\mu), \tag{7}$$

in which

$$P(t)=H^T(0)A(t)H^T(0)^{-1},\quad g(t)=H^T(0)f(t),$$
$$Q_i=H^T(0)B_iH^T(0)^{-1},\quad J_i=H^T(0)I_i,$$
$$\mathcal{F}(t,y,\mu)=H^T(0)F(t,H^T(0)^{-1}y,\mu),\qquad \mathcal{P}_i(y,\mu)=H^T(0)\Omega_i(H^T(0)^{-1}y,\mu).$$

Denote $y(t, \alpha) = H^T(0)x(t, \alpha)$, $\beta = (\beta_{k+1}, \ldots, \beta_n)$ and let $\nu(t) = y(t, \alpha, \beta)$ be a solution of system (7) with initial condition $\nu(0) = y(0, \alpha) + (0, \beta)$. Further, let $L(t) = H^{-1}(0)H(t)$, $L_1(t) = H^{-1}(0)H_1(t)$, let $L_2(t)$ be the matrix composed of the entries of the last $n - k$ columns and $n - k$ rows of the matrix $L(t)$, and let $L_3(t)$ be the matrix composed of the last $n - k$ rows of $L(t)$. Denote

$$U(\alpha,\beta,\mu)=\int_0^\omega L_1^T(t)\mathcal{F}(t,\nu(t),\mu)\,dt+\sum_{i=1}^p L_1^T(t_i)\mathcal{P}_i(\nu(t_i),\mu),$$
$$V(\alpha,\beta,\mu)=(L_2^T(\omega)-E)\beta-\mu\int_0^\omega L_3^T(t)\mathcal{F}(t,\nu(t),\mu)\,dt-\mu\sum_{i=1}^p L_3^T(t_i)\mathcal{P}_i(\nu(t_i),\mu).$$

Then the ω-periodicity condition for the solution $\nu(t)$ takes on the form of the equations

$$U(\alpha,\beta,\mu)=0, \tag{8}$$
$$V(\alpha,\beta,\mu)=0. \tag{9}$$

If in (9) one takes $\mu = 0$, we obtain $\beta = 0$, and then Eq. (8) has the form

$$U(\alpha, 0, 0) = \int_0^{\omega} L_1^T(t)\, \mathcal{F}(t, y(t, \alpha), 0)\, dt + \sum_{i=1}^{p} L_1^T(t_i)\, \mathcal{P}_i(y(t_i, \alpha), 0) = 0. \tag{10}$$

Let $\alpha_0 = (\alpha_1{}^0, \ldots, \alpha_k{}^0)$ be a solution of Eq. (10). Since the function U has continuous partial derivatives with respect to α_j, $j = 1, \ldots, k$, in a sufficiently small neighborhood of the point $(\alpha_0, 0, 0)$, it follows that under the assumption that

$$\det \left[\frac{\partial U}{\partial \alpha} \bigg|_{\alpha=\alpha_0} \right] \neq 0$$

the system of equations (8), (9) is solvable with respect to α and β so that the functions $\alpha_j(\mu)$ and $\beta_s(\mu)$, $j = 1, \ldots, k$, $s = k + 1, \ldots, n$ are continuous and $\alpha_j(\mu) \to \alpha_j{}^0$, $\beta_s(\mu) \to 0$ as $\mu \to 0$.

Thus, we established that for sufficiently small $|\mu|$ system (6) admits an ω-periodic solution, which for $\mu \to 0$ converges to the solution $x(t, \alpha_0)$ of system (4).

To complete the proof of the theorem it remains to return from system (6) to Eqs. (1).

2. Periodic Solutions of an Autonomous System. Consider the system

$$\frac{dx}{dt} = Ax + \mu F(x, \mu), \quad x \,\bar{\in}\, \Gamma_1, \quad \Delta x|_{x \in \Gamma_1} = W(x, \mu), \tag{11}$$

where $x \in R^n$, μ is a small parameter, F, $W \in C^{(1)}$, and A is a constant $n \times n$ matrix. Also, for fixed μ, Γ_1 is the surface in R^n given by the equation

$$\Phi(x) + \mu\Phi_1(x, \mu) = 0, \quad \Phi, \quad \Phi_1 \in C^{(1)}. \tag{12}$$

Suppose that the linear homogeneous equation corresponding to (11),

$$\frac{dx}{dt} = Ax, \tag{13}$$

admits k ($k < n$) linearly independent ω-periodic solutions φ_i, $i = 1, \ldots, k$. The general ω-periodic solution of this equation has the form

$$x(t, \alpha) = \alpha_1\varphi_1 + \ldots + \alpha_k\varphi_k, \quad \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_k).$$

Let Γ be the surface in R^n defined by the equation $\Phi(x) = 0$. Denote by $\tau_i(\alpha)$, $i = 1, \ldots, p$ the instants in the interval $[0, \omega]$ at which the solution $x(t, \alpha)$ meets the surface Γ, $0 < \tau_1(\alpha) < \tau_2(\alpha) < \ldots < \tau_p(\alpha) < \omega$. Suppose that for any α in a neighborhood of a point α_0, to be specified below, the solutions $x(t, \alpha)$ intersect the surface Γ nontangentially. To ensure this it suffices to assume that the relation

$$\left\langle \frac{\partial\Phi(x(\tau_i(\alpha), \alpha))}{\partial x}, \; Ax(\tau_i(\alpha), \alpha) \right\rangle \neq 0,$$

holds for all $i = 1, \ldots, p$, where $\langle , \rangle$ denotes the inner product in R^n. Let $H_1(t)$ denote the $n \times k$ matrix whose columns are linearly independent ω-periodic solutions ψ_j, $j = 1, \ldots, k$, of the system transposed to (13),

$$\frac{dx}{dt} = -A^T x. \tag{14}$$

Complete $H_1(t)$ by column-solutions ψ_j, $j = k + 1, \ldots, n$ so as to obtain a fundamental matrix of solutions $H(t)$. The substitution $y = H^T(0)x$, $t = \tau(1 + \mu\eta)$, where η is a function whose dependence on μ and on one of the coordinates of the vector α will be specified below, reduces system (11) to the equations

$$\frac{dy}{d\tau} = Qy + \mu(1+\mu\eta)\mathcal{F}(y,\mu) + \mu\eta Qy, \quad y \,\overline{\in}\, G_1, \tag{15}$$
$$\Delta y|_{y\in G_1} = \mu\mathcal{P}(y,\mu),$$

where

$$Q = H^T(0)\,AH^T(0)^{-1}, \quad \mathcal{F}(y,\mu) = H^T(0)\,F(H^T(0)^{-1}y,\mu),$$
$$\mathcal{P}(y,\mu) = H^T(0)\,\mathbb{W}(H^T(0)^{-1}y,\mu).$$

The set G_1 is given by the equation

$$\Phi(H^T(0)^{-1}y) + \mu\Phi_1(H^T(0)\,y,\mu) = 0. \tag{16}$$

One can verify that the functions figuring in Eqs. (15) and (16) satisfy the same differentiability conditions as those figuring in expressions (11) and (12). Denote $y(t,\alpha) = H^T(0)x(t,\alpha)$ and let $\nu(t) = y(t,\alpha,\beta,\mu,\eta)$ with $\beta = (\beta_{k+1},\ldots,\beta_n)$ be the solution of system (15) with initial condition $\nu(0) = y(0,\alpha) + (0,\beta)$. The ω-periodicity condition for the solution $\nu(t)$ is expressed by the equation

$$\begin{aligned} Z(\alpha,\beta,\mu,\eta) \equiv (L^T(\omega) - E)\,\nu(0) - \mu(1+\mu\eta)\int_0^\omega L^T(t)\,\mathcal{F}(\nu(t),\mu)\,dt - \\ - \mu\eta\int_0^\omega L^T(t)\,Q\nu(t)\,dt - \mu\sum_{i=1}^{p} L^T(\theta_i)\,\mathcal{P}(\nu(\theta_i),\mu) = 0, \end{aligned} \tag{17}$$

in which $\theta_i = \theta_i(\alpha,\beta,\mu,\eta)$ are the discontinuity points of $\nu(t)$, $L(t) = H^{-1}(0)H(t)$. Let $L_1(t) = H^{-1}(0)H_1(t)$, and let $L_2(t)$ be the matrix composed from the entries of the last $n-k$ columns and last $n-k$ rows of the matrix $L(t)$. Then using the properties of $H(t)$ we find that Eq. (17) decomposes into the following equations

$$U(\alpha,\beta,\mu,\eta) \equiv (1+\mu\eta)\int_0^\omega L_1^T(t)\,\mathcal{F}(\nu(t),\mu)\,dt + \eta\int_0^\omega L_1^T(t)\,Q\nu(t)\,dt + \sum_{i=1}^{p} L_1^T(\theta_i)\,\mathcal{P}(\nu(\theta_i),\mu) = 0, \tag{18}$$

$$(L_2^T(\omega) - E)\beta - \mu(1+\mu\eta)\int_0^\omega L_2^T(t)\,\mathcal{F}(\nu(t),\mu)\,dt - \mu\eta\int_0^\omega L_2^T(t)\,Q\nu(t)\,dt - \mu\sum_{i=1}^{p} L_2^T(\theta_i)\,\mathcal{P}(\nu(\theta_i),\mu) = 0. \tag{19}$$

If in (19) we set $\mu = 0$ we obtain $\beta = 0$. Then Eq. (18) takes on the form

$$U(\alpha,0,0,\eta) = 0. \tag{20}$$

Let $\alpha_0 = (\alpha_1{}^0,\ldots,\alpha_k{}^0)$, $\eta = \eta_0$ be a solution of Eq. (20). By Theorem 3 below, the solution $\nu(t)$ has B-derivatives with respect to α, β, μ, η in a sufficiently small neighborhood of the point $(\alpha_0, 0, 0, \eta_0)$.

From the fact that the right-hand side of the variational system depends continuously on the initial conditions and on the parameters it follows that the aforementioned B-derivatives depend continuously in the B-topology on α, β, μ, η. Consequently, the functions θ_i, $i = 1,\ldots,p$ are differentiable with respect to α, β, μ, η and hence the function $Z(\alpha,\beta,\mu,\eta)$ is differentiable in a sufficiently small neighborhood of the point $(\alpha_0, 0, 0, \eta_0)$. In this case the condition

$$\det\left[\left.\frac{\partial U(\alpha,\beta,\mu,\eta)}{\partial(\alpha_1,\ldots,\alpha_{j-1},\alpha_{j+1},\ldots,\alpha_k,\eta)}\right|_{(\alpha_0,\eta_0)}\right] \neq 0, \tag{21}$$

where j takes one of the values $1, 2, \ldots, k$, suffices to guarantee for small $|\mu|$ the existence of a solution $\alpha_1(\mu,\alpha_j),\ldots,\alpha_{j-1}(\mu,\alpha_j),\alpha_{j+1}(\mu,\alpha_j),\ldots,\alpha_k(\mu,\alpha_j),\beta(\mu,\alpha_j),\eta(\mu,\alpha_j)$ of Eqs. (18), (19) with the property that it tends to the values $\alpha = \alpha_0$, $\beta = 0$, $\eta = \eta_0$ when $\mu \to 0$, $\alpha_j \to \alpha_j{}^0$.

System (20) consists of k equations with respect to the (k + 1)-st unknown. Hence, one can choose from the very beginning $\alpha_k^0 = 0$. In view of the one-to-one correspondence between the vectors α and $\xi = (y_1(0, \alpha), \ldots, y_k(0, \alpha))$, this last condition means that ξ belongs to some fixed hyperplane in R^k.

Let us return to system (11) and write Eq. (20) in the form

$$\int_0^\omega H_1^T(t) F(x(t, \alpha), 0)\, dt + \eta \int_0^\omega H_1^T(t) Ax(t, \alpha)\, dt + \sum_{i=1}^p H_1^T(\tau_i(\alpha)) W(\tau_i(\alpha), 0) = 0. \tag{22}$$

From this and the continuous dependence of the solutions of the impulse system (11) on the initial conditions and the parameters we derive the following result.

THEOREM 2. Suppose system (11) satisfies the conditions discussed above and $\alpha = \alpha_0$, $\eta = \eta_0$ is a solution of system (22) with the property that condition (21) holds for at least one of the values 1, 2, ..., k. Then for sufficiently small $|\mu|$ system (11) admits a periodic solution of period ω' such that for $\mu \to 0$ the period ω' tends to the value ω, and the solution itself tends in the B-topology to $x(t, \alpha_0)$.

Remark. The function $x(t, \alpha_0)$ is a solution of the system (13) of ordinary differential equations, and as such is continuous. In order that the assertion concerning B-convergence be meaningful we assume that $x(t, \alpha_0)$ undergoes null jumps at the points $\tau_i(\alpha_0)$.

3. Differentiability of Solutions of an Autonomous Impulse System with Respect to Initial Conditions and Parameters. Suppose given the autonomous system

$$\frac{dx}{dt} = f(x, \mu), \quad x \bar{\in} \Gamma, \quad \Delta x|_{x \in \Gamma} = I(x, \mu), \tag{23}$$

where $x \in R^n$, $\mu \in R^m$, $f, I \in C^{(1)}$. For fixed μ the set Γ is a surface in R^n given by an equation $\Phi(x, \mu) = 0$, where $\Phi \in C^{(1)}$. Let $x_0(t) = x(t, t_0, x_0, \mu_0)$ be a solution to the Cauchy problem for system (23). Suppose that over the finite interval $K = [t_0, t_0 + T]$ the solution $x_0(t)$ hits the set Γ at instants τ_i, $i = 1, \ldots, p$, $t_0 < \tau_1 < \tau_2 < \ldots < \tau_p < t_0 + T$. Denote

$$f_\pm^i = f(x_0(\tau_i \pm), \mu_0), \quad I_{xi} = I'_x(x_0(\tau_i), \mu_0),$$
$$\Phi_{xi} = \Phi'_x(x_0(\tau_i), \mu_0), \quad A(t) = f'_x(x_0(t), \mu_0).$$

We shall assume that $\langle \Phi_{xi}, f_-^i \rangle \neq 0$, and denote

$$J_i^j = (f_+^i - (E + I_{xi}) f_-^i) \frac{\Phi'_{\mu_j}(x_0(\tau_i), \mu_0)}{\langle \Phi_{xi}, f_-^i \rangle} + I'_{\mu_j}(x_0(\tau_i), \mu_0),$$
$$g_j(t) = f'_{\mu_j}(x_0(t), \mu_0).$$

Further, let P_i denote matrices such that the following relations hold for all $z \in R^n$:

$$P_i z = I_{xi} z + (f_+^i - f_-^i - I_{xi} f_-^i) \frac{\langle \Phi_{xi}, z \rangle}{\langle \Phi_{xi}, f_-^i \rangle}.$$

Using the methodology developed in the papers [5-8] one can prove the validity of the following result.

THEOREM 3. Suppose system (23) and its solution $x_0(t)$ satisfy the conditions discussed above. Then in the interval K the solution $x_0(t)$ has:

1) B-derivatives $u_j(t)$ with respect to x_0^j, $j = 1, \ldots, n$, which are solutions of the linear system

$$\frac{du}{dt} = A(t) u, \quad t \neq \tau_i, \quad \Delta u|_{t=\tau_i} = P_i u$$

with initial conditions $u_j(t_0) = \underbrace{(0, \ldots, 0, 1}_{j}, 0, \ldots, 0)$;

2) B-derivatives $v_k(t)$ with respect to $\mu_0{}^k$, which are solutions of the system

$$\frac{dv}{dt} = A(t)v + g_k(t), \quad t \neq \tau_i, \quad \Delta v|_{t=\tau_i} = P_i v + J_i^k$$

with initial conditions $v_k(t_0) = 0$, $k = 1, \ldots, m$.

LITERATURE CITED

1. N. M. Krylov and N. N. Bogolyubov, Introduction to Nonlinear Mechanics [in Russian], Izd. Akad. Nauk Ukr. SSR, Kiev (1937).
2. Yu. A. Mitropol'skii, The Averaging Method in Nonlinear Mechanics [in Russian], Naukova Dumka, Kiev (1977).
3. A. M. Samoilenko and N. A. Perestyuk, Differential Equations with Impulse Action [in Russian], Vishcha Shkola, Kiev (1987).
4. I. G. Malkin, Some Problems of the Theory of Nonlinear Oscillations [in Russian], Gostekhizdat, Moscow (1956).
5. M. U. Akhmetov and N. A. Perestyuk, "On the motion with impulse action on surfaces," Izv. Akad. Nauk Kazakh. SSR, No. 1, 11-14 (1988).
6. M. U. Akhmetov and N. A. Perestyuk, "Periodic solutions of differential equations with impulse action on surfaces," in: Application of Asymptotic Methods in the Theory of Nonlinear Oscillations [in Russian], Inst. Mat. Akad. Nauk Ukrain. SSR, Kiev (1987). pp. 11-14.
7. M. U. Akhmetov and N. A. Perestyuk, "On the differentiable dependence of the solutions of impulse systems on initial data," Ukr. Mat. Zh., 41, No. 8, 1028-1033 (1989).
8. M. U. Akhmetov and N. A. Perestyuk, "Stability of periodic solutions of systems of differential equations with impulse action on surfaces," Ukr. Mat. Zh., 41, No. 12, 1596-1604 (1989).

AVERAGING IN PARABOLIC SYSTEMS SUBJECT TO WEAKLY DEPENDENT RANDOM ACTIONS. THE L_2-APPROACH

B. V. Bondarev

UDC 519.21

The first initial-boundary problem for a parabolic equation with a small parameter under external action described by some random process satisfying an arbitrary condition of weak dependence is considered. Averaging of the coefficients over a time variable is carried out. The existence of a generalized solution for the initial stochastic problem as well as for the problem with an "averaged" equation which turns out to be deterministic is assumed. Exponential bounds of the type of the well-known Bernstein inequalities for a sum of independent random variables are established for the probability of the deviation of the solution of the initial equation from the solution of the "averaged" problem.

Consider on $\Theta_{T/\varepsilon} = [0, T/\varepsilon] \times G$ the first initial-boundary problem

$$\frac{\partial U_\varepsilon}{\partial t} = \varepsilon[\mathscr{L}_{t,x} U_\varepsilon + A(t, x, U_\varepsilon) + \sigma(t, x, U_\varepsilon)\eta(t)],$$
$$U_\varepsilon(t, x)|_{t=0} = \varphi(x), \quad U_\varepsilon(t, x)|_{x \in \partial G} = \Phi(t, x). \tag{1}$$

Donetsk University.

Translated from Ukrainskii Matematicheskii Zhurnal, Vol. 43, No. 3, pp. 315-322, March, 1991. Original article submitted March 3, 1990.