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★Principles of discontinuous dynamical systems.

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Preface: "The main subject of this book is discontinuous dynamical systems. These have played an extremely important role theoretically, as well as in applications, for the last several decades. Still, the theory of these systems seems very far from being complete, and there is still much to do to make the application of the theory more effective. This is especially true of equations with trajectories discontinuous at moments that are not prescribed.

"The book is written not only on the basis of research experience but also, importantly, on the basis of the experience of teaching the course of Impulsive differential equations for about 10 years to the graduate students of mathematics. It is useful for a beginner as we try not to avoid any difficult instants in delivering the material. Delicate questions that are usually ignored in a research monograph are thoroughly addressed. The standard material on equations with fixed moments of impulses is presented in a compact and definitive form. It contains a large number of exercises, examples, and figures, which will aid the reader in understanding the enigmatic world of discontinuous dynamics. The following peculiarity is very important: the material is built on the basis of close parallelism with ordinary differential equations theory. For example, even higher order differentiability of solutions, which has never been considered before, is presented with a full definition and detailed proofs. At the same time, the definition of the derivatives as coefficients of the expansion is fruitfully used, which is very rare in the theory of ordinary differential equations. Moreover, the description of stability, continuous and differentiable dependence of solutions on initial conditions, and right-hand side, chaotic ingredients is given on a more strong functional basis than that of ordinary differential equations.

"The book is attractive to an advanced researcher, since a strong background for the future analysis of all theoretical and application problems is built. It will benefit scientists working in other fields of differential equations with discontinuities of various types, since it reflects the experience of the author in working on these subjects. We would like to emphasize that the basics of discontinuous flows are for the first time rigorously laid out so that all the attributes of dynamical systems are present. Hence, there is plenty of room for extending all the results of continuous, smooth and analytic dynamics to the systems with discontinuities.

"The content of the book is a good background for the application in vibromechanisms theory, mechanisms with friction, biology, molecular biology, physiology, pharmacology, secure communications, neural networks, and other real world problems involving discontinuities.

"Chapters 5–10 contain the core research contributions. Chapters 1–4 present preliminaries for the theory and elements of differential equations with fixed moments of discontinuity. Chapters 1–8 provide sufficient material for a standard one-semester graduate course. It is natural to finalize a general theory with more specific results. For this reason, in the last two chapters (9 and 10) we discuss Hopf bifurcation of periodic discontinuous solutions, Devaney's chaos, and the Shadowing property for discontinuous dynamical systems.

"The author expresses his gratitude to his students who have contributed to the

preparation of this book: Duygu Arugaslan, Cemil Buyukadali, Mehmet Turan, and Enes Yilmaz."

Contents: 1. Introduction; 2. Description of the system with fixed moments of impulses and its solutions; 3. Stability and periodic solutions of systems with fixed moments of impulses; 4. Basics of linear systems; 5. Nonautonomous systems with variable moments of impulses; 6. Differentiability properties of nonautonomous systems; 7. Periodic solutions of nonlinear systems; 8. Discontinuous dynamical systems; 9. Perturbations and Hopf bifurcation of a discontinuous limit cycle; 10. Chaos and shadowing; References; Index.