Other-Regarding Preferences in Organizational Hierarchies

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Abstract

In this paper, we provide new theoretical insights about the role of collusion in organizational hierarchies by combining the standard principal-supervisor-agent framework with a theory of social preferences. Extending Tirole's (1986) model of hierarchy with the inclusion of Fehr and Schmidt (1999) type of other-regarding preferences, the links between inequity aversion, collusive behavior and changes in optimal contracts are studied. It turns out that other-regarding preferences do change the collusive behavior among parties depending on the nature of both agent's and supervisor's other-regarding preferences. The most prominent impact is on the optimal effort levels. When the agent is inequity averse, the principal can exploit this fact to make agent exert higher effort level than she would otherwise. In order to satisfy the participation constraint of the supervisor, the effort level induced for the agent becomes lower when the supervisor is status seeker, and it is higher when the supervisor is inequity averse.

Keywords: Other-Regarding Preferences, Hierarchy, Collusive Behavior, Optimal Contract Design

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1 Introduction

Many models in economics use self-interest approach for the analysis of real-life problems. However, countless lab and field experiments in economics showed that self-interest approach may not be sufficient in explaining all of the observed behavior.[1] Moreover, these experiments highlighted the fact that many people have other-regarding preferences, and concern for another's situation is a motivating factor in most people's decision-making processes. Supporting the experiments, many theoretical papers showed that this behavior can be presented in a tractable way. We also follow this path and investigate how other-regarding preferences shape the optimal collusion-proof contract in the principal-supervisor-agent hierarchy. We show that the possibility of collusion between employees and wage payments in an optimal contract depend on the degree of social preferences. Moreover, the role of collusion can be ignored in an organization if the supervisor's degree of inequity aversion is above a certain threshold.

Collusive behavior is not a rare event in organizations. The fact that several layers of people interact with one another puts an emphasis on group gains as well as individual gains which open a way to forming coalitions among different parties. Hence, hierarchy design for an organization must consider the possibility of corruptive activities and aim to prevent them beforehand. To present a way to construct this kind of design, in his seminal work, Tirole (1986) puts sociological studies on corruption in hierarchies into a formal theoretical model by adding a supervisor layer between the standard principal-agent network. Although he acknowledges the fact that interaction between the supervisor and the agent affects the shape of their relationship, he still prefers to use self-interested parties in his theoretical framework.

The literature in the standard principal-agent contract theory also uses the assumption of self-interest. However, several papers have used the theories of social preferences in the

[1] See Ledyard (1995) and Fehr and Schmidt (2006) for public and private good environments respectively.

principal-agent theoretical framework.[2] On the other hand, these models have not been widely affiliated with the collusive behavior and optimal contracts in a three-level hierarchy, though Tirole (1986), for example, considers this type of hierarchies as a network of standard two-tier contracts.

We provide a theoretical framework to complement empirical studies on how social preferences in an organization affect collusion and behavior of the parties and hence optimal contract design. Several empirical studies (e.g., Agell and Lundborg (1995), Blinder and Choi (1990), Campbell and Kamlani (1997), Bewley (2002) and Gartenberg and Wulf (2017)) report that employees in an organization not only care about their own well-being but also take the well-being of their co-workers into consideration, and managers design contracts that avoid too much internal inequality. Pepper and Gore (2015) argue that an employee of an organization may take the compensation (rewards) of her peers, immediate subordinates or immediate superiors as a reference for her own rewards and compensation. Hence, it appears essential to apply other-regarding preferences into hierarchical models. By incorporating other-regarding preferences, it is possible to gain further and more realistic insights about collusive behavior and optimal contracts.

The first objective of this paper is to introduce other-regarding preferences for the supervisor and the agent and investigate its effect on the collusive behavior. The second one is to explore the changes in the optimal collusion-proof contract parameters: i) effort levels exerted by the agent; and ii) wages. We choose to use Fehr and Schmidt (1999) type of other-regarding utility function since it is simple, powerful and can easily be applied to the principal-supervisor-agent hierarchy. We investigate the case in which the supervisor is either inequity averse or status seeker, and the agent is only inequity averse since we assume that the agent's wage is always lower than the wage of the supervisor.

The main observation is that introducing other-regarding preferences changes the amount that a briber has to pay in order to persuade the other party and also the amount that she can

[2] See, among others, Itoh (2004), Rey Biel (2008), Neilson and Stowe (2010) and Koszegi (2014).

pay at most if she wants to form a coalition. The components of contracts (especially effort exerted by the agent) also change with other-regarding preferences. Though the ranking of both agent's and supervisor's utilities at different states does not differ from Tirole's (1986) model, wages and dispersion among wages at different states vary due to changes in collusion constraints shaped by social preferences. Furthermore, we find that inequity aversion results in wage contraction between layers of the hierarchy. Lastly, depending on the degree of social preferences, the principal can escape from the burden of preventing collusion between employees since the net benefit of collusion can be negative for employees with other-regarding preferences.

The next section introduces our model, which is basically Tirole's (1986) model with Fehr and Schmidt (1999) type of other-regarding preferences. Afterward, we analyze the principal's problem and investigate the structure of collusion-proof contracts when both supervisor and agent have other-regarding preferences. Section 3 contains our main results. We conclude the paper with a discussion and open questions for future research. All proofs are relegated to the appendix.

2 The Model

Our model is built on Tirole's (1986) three level (principal-supervisor-agent) hierarchy model. In order to show the effects of inequity aversion on collusion and optimal contracts, we use Fehr and Schmidt's (1999) theory of inequity aversion in the definition of both supervisor's and agent's utilities.[3]

The Parties: The only productive unit is the agent. The principal's profit depends on the agent's productive effort $e > 0$ and the productivity parameter θ in the environment according to the technology $x = \theta + e$.

The agent faces disutility coming from the exerted effort, and it is denoted by $g(e)$ in

[3] We refer the reader to Tirole (1986) and Fehr and Schmidt (1999) for discussion and justification of the environment and assumptions.

monetary terms where g is strictly convex, increasing in effort, and $g(0) = g'(0) = 0$. The principal pays a wage W to the agent.

The supervisor's main role is to monitor the agent and her environment, and then report the result of her inspection about the productivity level θ to the principal. The details about the supervisor's duty are explained later together with the definition of the hidden action and hidden information problem existing in our model. As in Tirole (1986), we impose a simple supervision technology in which the supervisor exerts no effort while observing the agent's productivity; thus, there is no monitoring cost for the supervisor. Principal pays a wage S to the supervisor.

The assumption about supervisor's and agent's utility functions is the main difference between our model and Tirole's (1986) model. This is actually our main contribution. The inequity aversion approach suggests that interacting participants in hierarchical environments do care about not only their own payoffs but also the payoff of the interacted parties. In addition, other-regarding preferences occur among people within similar social circles.[4] Although the supervisor and the agent are not at the same levels of the hierarchy, it is reasonable to assume that the employees see each other as co-workers; hence, we assume that other-regarding behavior is observed between the employees. Since the principal is in the owner role of the whole game, her total gain is not considered in the utilities of the employees, or the payoffs of the supervisor and the agent do not have any effect on the utility of the principal.

The agent's utility in our environment depends on her monetary income minus her effort cost $(W - g(e))$ and the cost of social comparison with the supervisor. To model inequity aversion, one should have to decide whether the comparison between payoffs of interacting parties should take into account only the difference in wages or the difference in wages after subtracting effort costs. Neilson and Stowe (2010) argue that employees tend to make direct wage comparisons without considering efforts. Moreover, according to Bartling and Siemens

[4]See, among others, Agell and Lundborg (1995), Blinder and Choi (1990), Campbell and Kamlani (1997), Bewley (2002) and Pepper and Gore (2015).

(2010), "*effort costs and wages accrue in different dimensions such that it is not obvious how they are aggregated. Wages instead are directly comparable and thus constitute the most salient reference point*". One additional argument may be that wages are probably harder to observe than efforts.[5] However, employees often reveal how much money they make to each other even when there is a secrecy of salaries as a company policy (Bewley (2002)) and informal agreement among employees to anonymously share their salary information is a common practice in many organizations even though employees sign contracts in which they agree not to share the information about their wages (Nickerson and Zenger (2008)). It is also reasonable to think that workers may not perfectly observe their co-workers' effort levels and effort costs. Extra effort and time may be needed to get this information. Hence, comparing effort costs is a more difficult task than gross wage comparison. We also know that people have behavioral tendencies to concentrate on comparing easy comparable things so they do not need to exert more mental effort to figure out relative differences, and they avoid costly relative comparisons when major differences are not easy to point out (Ariely (2009)). In addition, people tend to make mentally effortless intuitive comparisons by using "System 1" rather than "System 2" which requires mental effort for analytical thinking (Kahneman (2003)).

In our formulation of the utility functions, we assume that employees only compare wages as in Grund and Sliwka (2005), Dur and Glazer (2008), Kragl and Schim (2009), Bartling and Siemens (2010) and Cato (2013). That is, the agent's utility depends on her monetary payoff minus her effort cost ($W - g(e)$) and the difference in wages ($S - W$). The supervisor's utility depends on her monetary income and the difference in wages since there is no monitoring cost for the supervisor. We also assume that the supervisor gets a higher salary than the agent does in every state of information, i.e., $S > W$ in all states. This assumption corresponds to the most of wage settings in the real world hierarchies and simplifies the construction and solution of our model without weakening its applicability.

[5] We thank an anonymous referee for pointing this out.

Using Fehr and Schmidt's (1999) utility function with two inequity averse employees, we define the utility function of the agent as:

$$U(W - g(e) - \lambda_A(S - W)),$$

and the supervisor's utility function as:

$$V(S - \lambda_S(S - W)),$$

where λ_A and λ_S represent inequity aversion parameters of the agent and the supervisor respectively. Since $S > W$ at all states, $\lambda_A > 0$ (the agent is inequity averse and dislikes being behind) and $\lambda_S < 1$ (either the supervisor is inequity averse, $0 < \lambda_S < 1$, or the supervisor loves being ahead and she is a status-seeker, $\lambda_S < 0$). As λ_A increases the agent becomes more inequity averse, i.e, becomes more sensitive to being behind. If $0 < \lambda_S < 1$, increase in λ_S makes supervisor more inequity averse and more sensitive to being ahead. On the other hand, given that $\lambda_S < 0$, as λ_S decreases the supervisor becomes more status-seeker and thrives more from being ahead. The case where $\lambda_A = 0$ and $\lambda_S = 0$ represents our benchmark case, as in Tirole (1986). Both U and V are differentiable, strictly concave and increasing Von Neumann Morgenstern utility functions with $U'(0) = \infty$ and $V'(0) = \infty$. We use expected utilities for both supervisor and agent in the analysis of our model since there are four different states of nature. The expected utility of the agent is $EU(W - g(e) - \lambda_A(S - W))$ and the expected utility of the supervisor is $EV(S - \lambda_S(S - W))$.

The supply of supervisors and agents is competitive, and agents have reservation wages W_0 with reservation utility $\overline{U} \equiv U(W_0)$, while supervisors have reservation wages S_0 with reservation utility $\overline{V} \equiv V(S_0)$. The participation (individual rationality) constraints for the agent and the supervisor can be written as:

$$EU(W - g(e) - \lambda_A(S - W)) \geq \overline{U},$$

$$EV(S - \lambda_S(S - W)) \geq \overline{V}.$$

The last party of this organizational hierarchy is the principal. She assigns agents to the work project and offers contracts to both supervisor and agent. The principal is risk-neutral and her expected utility is defined as:

$$E(x - S - W) = E(\theta + e - S - W).$$

Hidden Information Problem: There are two productivity levels in the working environment, low state of productivity $\underline{\theta}$ and high state of productivity $\overline{\theta}$, where $0 < \underline{\theta} < \overline{\theta}$, and $\Delta\theta = \overline{\theta} - \underline{\theta}$. The productivity levels ($\underline{\theta}$, and $\overline{\theta}$) and their probability of occurrence are common knowledge.

The agent becomes aware of the productivity level in the environment after signing a contract and determines her effort level according to the realized productivity. However, the supervisor sometimes fails to observe the productivity level. When the supervisor monitors the agent, one of the four following states of nature can arise:

State 1: Both agent and supervisor observe low level of productivity $\underline{\theta}$.

State 2: The agent observes $\underline{\theta}$. However, the supervisor fails to observe the current productivity level.

State 3: The agent observes high level of productivity $\overline{\theta}$. However, the supervisor fails to observe the current productivity level.

State 4: Both agent and supervisor observe $\theta = \overline{\theta}$.

Each state of nature has a probability of occurrence p_i where $\sum_{i=1}^{i=4} p_i = 1$.

Lastly, it is assumed that the agent has information on whether the supervisor observed the productivity level successfully or not. However, the agent cannot report supervisor's monitoring activity to the principal. The information structure becomes poorer as we go through the upper levels of the hierarchy. Moreover, the supervisor, as well as the principal, cannot observe the level of effort.

Timing: First, the principal offers contracts to both parties. Wages, S and W, are specified in these contracts as functions of observable and contractible variables which are the output, x, the report of the supervisor on the current productivity level, r, and both supervisor's and agent's inequity aversion parameters, λ_S and λ_A, which are assumed to be common knowledge for all parties in the organization. Both supervisor's and agent's wages become common knowledge to all parties when contracts are offered.

If contracts are accepted and before the uncertainty is resolved, i.e. parties do not know the level of θ, the supervisor and the agent move to a collusion stage and try to decide on side transfers.[6] Similar to the main contract offered by the principal, a side transfer is also a function of output, x, supervisor's report on productivity level, r, and inequity aversion parameters of the colluding parties, λ_A, and λ_S. Side transfers are not observable by the principal.

In the next step, the agent learns the productivity level in the environment, and then she chooses her effort level. This implies that the profit is also realized. On the other hand, the supervisor may or may not observe the productivity level. She prepares her report and presents it to the principal. If she fails to observe θ, her report is considered empty, $r = \emptyset$. In the case where the supervisor observes the productivity level successfully, she has the option to report her monitoring in a truthful manner or to hide the true information and give an empty report, i.e., the supervisor's report is $r \in \{\theta, \emptyset\}$.[7] One of the four states is realized after the reporting stage.

The final step is the execution of contracts. The principal pays S and W, after she learns the output level and the report. Moreover, if the employees decide to form a coalition at the collusion stage, side transfers are allocated.

First Best Solution with Self-Interested Parties (Tirole (1986)): In order to use

[6] We refer the reader to Tirole (1986) for detailed information on possible state misrepresentations and conditions for signing a side contract between the employees.

[7] Throughout the discussion about the context of the supervisor's report, we assume that when the supervisor observes the level of productivity in the environment, her report is considered as credible by the principal. On the other hand, the agent cannot make verifiable and credible announcements about the productivity level.

the results as a reference point later, consider the case in which the principal can observe the productivity level perfectly (no hidden information problem), and the effort exerted by the agent (no hidden action problem). Moreover, all employees in the hierarchy are self-interested utility maximizers ($\lambda_A = 0$, $\lambda_S = 0$).

In this case, the principal does not need a supervisory duty. We can find the optimal effort level e^* for the agent by solving the following optimization problem:

$$\underset{e}{max}\{\theta + e - W_0 - g(e)\}.$$

It is easy to see that $g'(e^*) = 1$ for both $\underline{\theta}$ and $\overline{\theta}$. Moreover, the wage paid to the agent is $W = W_0 + g(e^*)$ in all states.

3 Analysis of the Principal's Problem

The rest of our analysis follows the methodology below:

1) All of the post-side-transfer allocation constraints (individual rationality constraints, incentive compatibility constraints and no collusion constraints) are introduced to the principal's problem of optimal contract design.

2) The principal's net expected profit is maximized subject to the given constraints.

3) Results are analyzed and compared with the benchmark case where all employees are self-interested, i.e., $\lambda_A = 0$, and $\lambda_S = 0$ (Tirole, 1986).

First, note that the supervisor's wage is always higher than the agent's wage since the former is at the upper level of the hierarchy ($S_i > W_i$ for all $i \in \{1, 2, 3, 4\}$). As a result, the agent can only be inequity averse ($\lambda_A > 0$). On the other hand, the supervisor can show two different behaviors. She may be either inequity averse ($1 > \lambda_S > 0$) and feels bad about the fact that she is earning a higher wage than her co-worker, or status-seeker ($\lambda_S < 0$) and thrives from being ahead which is a sign of her position (status) against the agent.

We can now define the constraints for the principal's optimal contract design problem.

The participation constraints for the supervisor and the agent must be satisfied so that the main contract is accepted by both parties in the first place. The participation (individual rationality) constraints for the supervisor and agent are defined as follows:

$$(SPC): \quad EV(S-\lambda_S(S-W)) = \sum_i p_i V(S_i - \lambda_S(S_i - W_i)) \geq \overline{V} \equiv V(S_0),$$

$$(APC): \quad EU(W-g(e)-\lambda_A(S-W)) = \sum_i p_i U(W_i - g(e_i) - \lambda_A(S_i - W_i)) \geq \overline{U} \equiv U(W_0).$$

There is a hidden action problem in our model since the principal and supervisor cannot observe the effort. In states 1 and 4, the principal has the knowledge about the productivity levels. Hence, she can estimate the effort level using the output. However, this is not the case in states 2 and 3. When state 3 is realized, the agent can claim that it is actually state 2, and the profit of the principal is achieved on the low level of productivity $\underline{\theta}$ with the hard work of the agent even though the true state is actually $\overline{\theta}$. With her false information, she is able to exert less effort $e_2 - \Delta\theta$ instead of e_2 but earn the wage W_2 as if she exerts e_2. The principal must provide necessary incentives to make agent exert a higher level of effort when the supervisor's monitoring fails. The incentive compatibility for the agent is defined as follows:

$$(AIC): \quad W_3 - g(e_3) - \lambda_A(S_3 - W_3) \geq W_2 - g(e_2 - \Delta\theta) - \lambda_A(S_2 - W_2).$$

In order to prevent coalition between the supervisor and the agent, the principal must arrange the payoffs of both parties such that the total amount that the briber has to pay for a potential collusion must not be lower than how much she can actually pay. The principal tries to increase the amount of minimum side transfer at which the bribed party is indifferent between accepting and rejecting the bribe offer, and/or decrease the maximum amount of side payment the briber can provide without making herself worse off than the alternative case where she stays loyal to the true state. Collusion constraints can be seen as a formalization of these two strategies.

For a coalition hiding the low productivity in state 1, maximum side transfer must satisfy $W_2 - t_{max,1} - g(e_2) - \lambda_A(S_2 + t_{max,1} - W_2 + t_{max,1}) = W_1 - g(e_1) - \lambda_A(S_1 - W_1)$ such that it is not rational for the agent to offer more than $t_{max,1}$ when she wants to make supervisor provide false report. Moreover, the agent has to transfer at least $t_{min,1}$, satisfying $S_2 + t_{min,1} - \lambda_S(S_2 + t_{min,1} - W_2 + t_{min,1}) = S_1 - \lambda_S(S_1 - W_1)$, to the supervisor so that the latter is indifferent to the bribe offer. Then, in order to prevent collusion, the principal must arrange the main contract such that $t_{min,1} \geq t_{max,1}$ (CIC1, the first collusion constraint).

For the second collusion possibility, hiding the truth about the high productivity environment in state 4, the maximum side transfer paid by the agent must satisfy $W_3 - t_{max,2} - g(e_3) - \lambda_A(S_3 + t_{max,2} - W_3 + t_{max,2}) = W_4 - g(e_4) - \lambda_A(S_4 - W_4)$. In this case, the supervisor gets at least $t_{min,2}$, satisfying $S_3 + t_{min,2} - \lambda_S(S_3 + t_{min,2} - W_3 + t_{min,2}) = S_4 - \lambda_S(S_4 - W_4)$, to accept the side transfer. Then, the principal must arrange the main contract such that $t_{min,2} \geq t_{max,2}$ (CIC2, the second collusion constraint).

It is easy to see that it is impossible for the agent to bribe the successful supervisor when $\lambda_S \to 0.5$. The collusion constraints representing these cases $(CIC1)$ and $(CIC2)$ are dropped from the principal's problem. When the supervisor's sensitivity to wage inequality is $\lambda_S = 0.5$, she does not accept any coalition offer. It is because of the fact that there exists a certain value for λ_S where the disutility coming from accepting the bribe and increasing the inequality completely offsets the monetary gain of the corresponding side transfer. Moreover, for the values above this threshold $(\lambda_S > 0.5)$, the disutility from increasing inequality dominates the side payment's monetary payoff when the agent comes with a bribe offer. This implies that the role of collusion in the organizational hierarchy can be ignored if the supervisor's degree of inequity aversion is above the certain threshold which is 0.5 in our model. Therefore, we assume that $0 < \lambda_S < 0.5$ for the rest of the paper. Given this observation, we can now state the first two collusion constraints:

$$(CIC1): \quad \frac{S_1 - \lambda_S(S_1 - W_1)}{1 - 2\lambda_S} + \frac{W_1 - g(e_1) - \lambda_A(S_1 - W_1)}{1 + 2\lambda_A} \geq \frac{S_2 - \lambda_S(S_2 - W_2)}{1 - 2\lambda_S} + \frac{W_2 - g(e_2) - \lambda_A(S_2 - W_2)}{1 + 2\lambda_A},$$

$$(CIC2): \quad \frac{S_4 - \lambda_S(S_4 - W_4)}{1 - 2\lambda_S} + \frac{W_4 - g(e_4) - \lambda_A(S_4 - W_4)}{1 + 2\lambda_A} \geq \frac{S_3 - \lambda_S(S_3 - W_3)}{1 - 2\lambda_S} + \frac{W_3 - g(e_3) - \lambda_A(S_3 - W_3)}{1 + 2\lambda_A}.$$

Note that, for the possibility of collusion in states 1 and 4, paying a side transfer not only decreases the agent's monetary payoff but also makes her feel worse by increasing the inequality. Therefore, the maximum side transfer the agent can provide is reduced for an inequity averse agent in comparison to the self-interested one. On the other hand, paying a side transfer to the status-seeker supervisor not only increases her monetary payoff directly but also makes her feel better by increasing the wage difference between her and the agent, i.e., by increasing her position (status) against that of the agent. Hence, a side transfer may make supervisor more prone to accept a bribe offer in comparison to the benchmark case. Things are completely different for an inequity averse supervisor. Accepting a side transfer makes her feel worse due to increase in inequality and negates the positive effect on the payoff coming from the monetary side payment. Thus, a side transfer's impact on making the supervisor accept a bribe offer is reduced in comparison to the benchmark.

Another coalition may occur in state 3 when the supervisor wants to bribe the agent to make her imitate the low-productivity agent and accept the incentive scheme of state 2. In this case, the minimum side transfer must satisfy $W_2 + t_{min,3} - g(e_2 - \Delta\theta) - \lambda_A(S_2 - t_{min,3} - W_2 - t_{min,3}) = W_3 - g(e_3) - \lambda_A(S_3 - W_3)$ so that the agent falsely claims that it is state 2. Moreover, maximum side transfer must satisfy $S_2 - t_{max,3} - \lambda_S(S_2 - t_{max,3} - W_2 - t_{max,3}) = S_3 - \lambda_S(S_3 - W_3)$ such that it is not rational for the supervisor to offer more than $t_{max,3}$ at most without making herself worse off than the no coalition case. Hence, in order to prevent collusion, the optimal contract must be arranged such that $t_{min,3} \geq t_{max,3}$ (CIC3, the third collusion constraint). That is,

$$(CIC3): \quad \frac{S_3 - \lambda_S(S_3 - W_3)}{1 - 2\lambda_S} + \frac{W_3 - g(e_3) - \lambda_A(S_3 - W_3)}{1 + 2\lambda_A} \geq \frac{S_2 - \lambda_S(S_2 - W_2)}{1 - 2\lambda_S} + \frac{W_2 - g(e_2 - \Delta\theta) - \lambda_A(S_2 - W_2)}{1 + 2\lambda_A}.$$

It is important to note that paying a side transfer to an inequity averse agent not only increases her monetary payoff but also makes her feel better by decreasing the inequality. Thus, in comparison to the self-interested agent, a smaller side transfer may make the inequity averse agent accept a bribe offer. On the other hand, in this case, paying a side transfer to the agent not only decreases the monetary payoff of the status-seeker supervisor but also makes her feel worse due to the disutility coming from reduction in her relative status against that of the agent. Thus, the maximum amount of side transfer the supervisor can provide is reduced for the status-seeker supervisor in comparison to the benchmark case. However, a side transfer to an agent makes an inequity averse supervisor feel better due to the decrease in inequality and dampens the negative effect of monetary loss. Hence, the maximum amount of side transfer is increased for an inequity averse supervisor in comparison with the benchmark.

We now solve the optimal contract design problem for the principal. By choosing S_i, W_i and e_i, the principal wants to maximize her expected utility, $\sum_i p_i(\theta_i + e_i - S_i - W_i)$, subject to the constraints. That is, we need to solve the following problem:

$$\underset{(S_i, W_i, e_i)}{max} \sum_i p_i(\theta_i + e_i - S_i - W_i)$$

subject to

$$(SPC), (APC), (AIC), (CIC1), (CIC2), and\ (CIC3).$$

In order to expose the impact of the other-regarding preferences, we first present the benchmark results by setting λ_A and λ_S equal to zero. This case corresponds to the problem tackled by Tirole (1986).

Theorem 1 ***(Tirole (1986))*** *The solution to the principal's problem with self-interested supervisor and agent (benchmark case) has the following properties:*

a) $S_4^B > S_1^B > S_2^B = S_3^B$

b) $W_3^B - g(e_3^B) > W_4^B - g(e_4^B) > W_1^B - g(e_1^B) > W_2^B - g(e_2^B)$ *and* $W_3^B > W_4^B > W_1^B > W_2^B$

c) $S_4^B + W_4^B = S_3^B + W_3^B$

d) $e_1^B = e_3^B = e_4^B = e^* > e_2^B$

e) All the constraints in the benchmark problem, except $(CIC1^B)$*, are binding.*[8]

The following result shows the effects of other-regarding preferences on the design of optimal collusion-proof contract.

Proposition 1 Effort levels in the optimal collusion-proof contract with other-regarding employees have the following properties:

a) $e_1 = e_3 = e_4 = \frac{1+\lambda_A-\lambda_S}{1-2\lambda_S} > e_2$

b) $\frac{\partial e_1}{\partial \lambda_A} = \frac{\partial e_2}{\partial \lambda_A} = \frac{\partial e_3}{\partial \lambda_A} = \frac{\partial e_4}{\partial \lambda_A} > 0$ and $\frac{\partial e_1}{\partial \lambda_S} = \frac{\partial e_2}{\partial \lambda_S} = \frac{\partial e_3}{\partial \lambda_S} = \frac{\partial e_4}{\partial \lambda_S} > 0$

c) If the inequity aversion of the agent dominates the status-seeking behavior of the supervisor (i.e., $\lambda_A > -\lambda_S$), then $e_1 = e_3 = e_4 > e^*$ and $e_2 > e_2^B$. There are also threshold values $\lambda_A(\Delta\theta, \lambda_S)$ and $\lambda_S(\Delta\theta, \lambda_A)$ such that $e_2 = e^*$.

If the status-seeking behavior of the supervisor dominates the inequity aversion of the agent (i.e., $\lambda_A < -\lambda_S$), then $e_2 < e_1 = e_3 = e_4 < e^*$ and $e_2 < e_2^B$.

If $\lambda_A = -\lambda_S$, then $e_1 = e_3 = e_4 = e^* > e_2$ and $e_2 = e_2^B$.

Proposition 1.a. states that the effort induced by the principal is lower in state 2 when the productivity is low, $\underline{\theta}$, and the supervisor's report is empty, $r = \phi$. This result is not because of the measures taken to prevent collusive behavior but to provide the necessary incentive to the agent not to imitate the low-productivity agent, and hence the agent exerts the effort level specified by the optimal contract for the high-productivity agent in state 3. This implies that the principal induces a lower effort level in state 2 so that she can

[8]The superscript B denotes the benchmark values for our problem.

reduce the amount of wage paid in that state, W_2, when the supervisor fails to monitor the agent. Therefore, the attractiveness of imitating the low-productivity agent is removed by the optimal contract.

The first inequality in Proposition 1.b. implies that the effort levels induced by the principal increase in all states as the agent cares more about the wage inequality. This is due to the fact that an inequity averse agent does not enjoy being behind and her wage increases with her effort level. To exploit this fact, the principal moderates the increasing cost of effort with the decrease in wage inequality. As the agent becomes more sensitive to being behind, it is easier to offset the cost of extra effort which means that the principal can induce higher levels of effort in all states. Therefore, in terms of effort levels, the principal is more likely to choose an agent with higher inequity aversion sensitivity since a higher level of effort leads to a higher level of output.

The second part of Proposition 1.b. is connected to the following two facts: i) the inequity averse supervisor does not enjoy being ahead, and the principal increases the wage of the agent in order to reduce wage inequality and satisfy the supervisor's participation constraint. A rise in the wages persuades the agent to exert a higher level of effort. The increase in the agent's wage is somewhat compensated with a higher effort level and a higher level of output; ii) as the supervisor becomes more inequity averse the increase in agent's wages must be higher. This implies that the principal should induce higher effort levels.

Note that above the optimal effort level, the additional wage necessary to increase the agent's effort is higher than the increase in the output coming from the additional effort. This and stated facts in the previous paragraph imply that having inequity averse rather than self-interested supervisor may be less beneficial for the principal, and it becomes more disadvantageous as the supervisor becomes more sensitive to wage inequality. Even if this is the case, increasing the agent's wage is always better than increasing the supervisor's wage solely to compensate for the disutility coming from wage inequality. This result is the implication of the fact that the supervisor is not a productive unit in our model.

A status-seeker supervisor enjoys being ahead since she sees wage inequality as a sign of status. Then the principal decreases the agent's wage and hence increases wage inequality. Reduction in wages implies that the agent now should exert a lower level of effort. As the supervisor becomes more sensitive to her sign of status, the agent's wage and consequently the effort level induced fall further. It is not unreasonable to think that inducing less effort, in this case, can be seen as a lost opportunity. However, we want to remark that the supervisor earns a wage without producing anything and lowering the agent's wage also opens a path in which the principal can also reduce the status-seeker supervisor's wage.

Proposition 1.c. is directly related to Proposition 1.b. because the self-interested agent and supervisor have zero sensitivity to wage inequality. When all employees are inequity averse, the principal can induce higher effort levels in every state relative to the benchmark case. In the case where the hierarchy has a status-seeker supervisor and $\lambda_A > -\lambda_S$, the need for extra wage (or extra effort) to reduce inequality has a priority over taking advantage of status-seeking behavior of the supervisor. As a result, we observe higher levels of effort than the benchmark in all states again. In states 1, 3 and 4, the principal induces more than optimal effort e^*. If the cost moderation effect of reducing wage inequality combined with the necessary change in agent wages to satisfy the other-regarding supervisor's participation constraint dominates the need for inducing a lower effort level (or a lower wage in state 2 to satisfy (AIC)), the principal can also induce a higher effort level than the benchmark effort level in state 2. Since higher the difference between high and low productivity levels, lower the effort level induced in state 2; the principal must employ an agent and/or a supervisor with a higher level of inequity aversion sensitivity if she wants to induce an effort level higher than the benchmark effort level in state 2.

In the case where status-seeking behavior of the supervisor dominates the inequity aversion of the agent ($\lambda_A < -\lambda_S$), the need for an extra wage (or extra effort) to reduce inequality is inferior to the exploitation of status-seeking behavior of the supervisor, and the reduction of effort levels are induced in the contract. The principal has to induce an effort level less than e^* in all states, and all effort levels are lower than the benchmark effort levels.

Lastly, the need for a decrease in the agent's wage and effort levels in order to benefit from exploiting status-seeking behavior of the supervisor offsets the necessity of higher wages and hence effort levels to reduce wage inequality if $\lambda_A = -\lambda_S$. In this case, we observe exactly the same results as in the benchmark case.

Proposition 2 All of the constraints, except $CIC1$, introduced to the principal's problem with other-regarding employees have strictly positive shadow prices, i.e. they are binding.

This result is identical to Theorem 1.e in the benchmark case (Tirole (1986)). All types of the agent always prefer to reveal that the reason for a lower level of output at state 2 is the low-productivity environment. In this case, the supervisor also supports the agent and acts as an advocate for her by reporting the true state of nature.

Proposition 3 Wages in the optimal collusion-proof contract with other-regarding employees have the following properties:

a) $S_4 - \lambda_S(S_4 - W_4) > S_1 - \lambda_S(S_1 - W_1) > S_3 - \lambda_S(S_3 - W_3) = S_2 - \lambda_S(S_2 - W_2)$

b) $W_3 - g(e_3) - \lambda_A(S_3 - W_3) > W_4 - g(e_4) - \lambda_A(S_4 - W_4) > W_1 - g(e_1) - \lambda_A(S_1 - W_1) > W_2 - g(e_2) - \lambda_A(S_2 - W_2)$

c) $\frac{S_4 - \lambda_S(S_4 - W_4)}{1 - 2\lambda_S} + \frac{W_4 - \lambda_A(S_4 - W_4)}{1 + 2\lambda_A} = \frac{S_3 - \lambda_S(S_3 - W_3)}{1 - 2\lambda_S} + \frac{W_3 - \lambda_A(S_3 - W_3)}{1 + 2\lambda_A}$

Proposition 3.a. shows the ranking of supervisor's possible utility levels in all states. The ranking is the same as the benchmark case. Note that wages alone can represent the utilities in the benchmark case. In our environment with an other-regarding supervisor, the principal has to consider both agent's and supervisor's wages at the same time. The supervisor gets the lowest payoff when she fails to report the productivity level. Setting $S_3 - \lambda_S(S_3 - W_3)$ lower than $S_4 - \lambda_S(S_4 - W_4)$ reduces the ability of the high-productivity agent to bribe the successful supervisor in order to misreport state 4 as state 3. On the other hand, setting $S_3 - \lambda_S(S_3 - W_3)$ too low results in another possibility of coalition in which unsuccessful supervisor persuades the high-productivity agent to mimic the low-productivity agent. This opportunity for collusion is eliminated by designing the contract such that $S_3 - \lambda_S(S_3 - W_3) = S_2 - \lambda_S(S_2 - W_2)$. Finally, the principal sets $S_1 - \lambda_S(S_1 - W_1) > S_2 - \lambda_S(S_2 - W_2)$ which rewards the successful supervisor and at the same time serves to satisfy her participation constraint.

In the benchmark case, the principal only considers the monetary payoff and exerted effort by the agent. When the agent has other-regarding preferences, the principal must also take into consideration the inequality between employees. Proposition 3.c. is a direct consequence of the collusion constraint ($CIC2$) and the fact that $e_3 = e_4$. Although the employees' total utility is identical in states 3 and 4, the individual payoffs are not identical in these states: $S_4 - \lambda_S(S_4 - W_4) > S_3 - \lambda_S(S_3 - W_3)$ and $W_3 - g(e_3) - \lambda_A(S_3 - W_3) > W_4 - g(e_4) - \lambda_A(S_4 - W_4)$. The supervisor's monitoring fails in state 3 and hence the high-productivity agent has an option to claim that she is working under low productivity environment. In order to prevent this, a higher payoff must be guaranteed for the agent in state 3. That is, $W_3 - g(e_3) - \lambda_A(S_3 - W_3) > W_2 - g(e_2) - \lambda_A(S_2 - W_2)$. On the other hand, optimal insurance for the agent tells us that her payoff in state 4 should be lower than the one in state 3. This gives a direct incentive to the agent to bribe the supervisor in state 4. Hence, the supervisor must achieve $S_4 - \lambda_S(S_4 - W_4) > S_3 - \lambda_S(S_3 - W_3)$ so that the principal can prevent collusion between employees. The difference between states 4 and 3 can be viewed as a cost of learning the true productivity level.

Although the ranking of utilities is not different than the benchmark case, the level of wages definitely changes according to the degree of other-regarding preferences. We need to specify an explicit formula for the utility functions in order to find by how much, and in which direction the wages differ from the benchmark wages. Thus, it may or may not be beneficial to employ an agent and/or a supervisor with other-regarding preferences depending on the utility functions of employees. Furthermore, we expect that the dispersion between wages at different states can become wider or narrower in comparison to the benchmark case since adjusting wages is one of the tools to prevent collusion. The effectiveness of this tool may be reduced or enhanced when the parties have other-regarding preferences. The principal may need a further increase or decrease in wages to satisfy the collusion constraints with other-regarding preferences.

4 Discussion and Concluding Remarks

Many people act as if they have other-regarding preferences in different economic environments, and concern for another's situation is a motivating factor in their decision-making processes in most situations. In this paper, we implement Fehr and Schmidt's (1999) distributional approach for other-regarding preferences into Tirole's (1986) three-level hierarchy model. Our aim is to analyze the effects of having other-regarding employees on collusive behavior in organizations.

Other-regarding preferences affect collusive behavior. Not only does it change the tendency of employees to offer a bribe or receive a bribe, but also it influences the effectiveness of the principal's tools that are used to prevent collusion between employees. Moreover, in the case where the supervisor's degree of inequity aversion reaches a certain threshold, she does not take any bribe offer coming from the agent. This implies that the role of collusion in organizational hierarchies can be ignored if the supervisor's degree of inequity aversion is above a certain threshold.

For the components of the optimal contract, the most prominent impact of other-regarding preferences is on effort levels. When the agent is inequity averse, the principal can exploit this fact to make agent exert a higher level of effort than she would otherwise. In order to satisfy the participation constraint of the supervisor, the effort level induced for the agent becomes lower when the supervisor is a status seeker, and it is higher when the supervisor is inequity averse. The optimal contract induces more-than-optimal effort when all employees are inequity averse or inequity aversion of the agent dominates the status-seeking behavior of the supervisor, less-than-optimal effort when the status-seeking behavior of the supervisor dominates the inequity aversion of the agent and optimal effort level when these two are equal.

Although the ranking of utilities in different states does not differ for self-interested and other-regarding employees, the ranking of their wages in different states is not identical. An explicit form of the utility functions is needed to show the exact change in wages. In addition to changes in the ranking, the dispersion between wages at different states can become wider or narrower in comparison to the benchmark case. Change in dispersion has consequences for risk-averse parties since they do not enjoy the cases in which their possible wages are highly different from their reservation wages.

We show that using other-regarding preferences in the design of optimal collusion-proof contracts generates new insights on the role of collusion in organizations. One natural direction for future research includes the impact of other-regarding preferences on different types of hierarchies. This paper investigates the effects of other-regarding parties in a model of principal-supervisor-agent hierarchy. We think that there is a great scope for implementing other-regarding preferences to other organizational hierarchies as in Bac (1996). Moreover, we assume that the degree of inequity aversion is common knowledge for simplicity. A further research with uncertain inequity aversion parameters seems like an interesting path to follow. Lastly, a further research on this topic is adding productive duties to the supervisor and making monitoring technology costly as in Bac and Kucuksenel (2006). Note that the principal always tries to change the effort level exerted by the agent since she is the only productive unit in the hierarchy. This limits the options that principal has to prevent collusion. Adding costly effort for the supervisor not only increases chances of the principal to satisfy the needs of other-regarding parties but also introduces a possibility of ex-ante collusion between employees. Investigating the changes in optimal contracts and collusive behavior for this new case may also be worthwhile.

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Appendix

Lagrangian for the solution of the principal's problem with other-regarding parties is:

$L = \sum_i p_i(\theta_i + e_i - W_i - S_i) + \nu(\sum_i p_i V(S_i - \lambda_S(S_i - W_i)) - \overline{V})$
$+\mu(\sum_i p_i U(W_i - g(e_i) - \lambda_A(S_i - W_i)) - \overline{U})$
$+\gamma(W_3 - g(e_3) - \lambda_A(S_3 - W_3) - W_2 + g(e_2 - \Delta\theta) + \lambda_A(S_2 - W_2))$
$+\psi((S_4 - \lambda_S(S_4 - W_4)(1 + 2\lambda_A) + (W_4 - g(e_4) - \lambda_A(S_4 - W_4))(1 - 2\lambda_S) - (S_3 - \lambda_S(S_3 - W_3))(1 + 2\lambda_A) - (W_3 - g(e_3) - \lambda_A(S_3 - W_3))(1 - 2\lambda_S))$
$+\pi((S_3 - \lambda_S(S_3 - W_3)(1 + 2\lambda_A) + (W_3 - g(e_3) - \lambda_A(S_3 - W_3))(1 - 2\lambda_S) - (S_2 - \lambda_S(S_2 - W_2))(1 + 2\lambda_A) - (W_2 - g(e_2 - \Delta\theta) - \lambda_A(S_2 - W_2))(1 - 2\lambda_S))$

Note that we first ignore $(CIC1)$; we are going to show that the solution satisfies $(CIC1)$.

Taking the derivatives of the Lagrangian above with respect to S_i, W_i, e_i results in following FOCs:

$$\nu V'(S_1 - \lambda_S(S_1 - W_1)) = \frac{1}{1 - \lambda_S} + \mu\frac{\lambda_A}{1 - \lambda_S}U'(W_1 - g(e_1) - \lambda_A(S_1 - W_1)) \tag{1}$$

$$\nu V'(S_2 - \lambda_S(S_2 - W_2)) = \frac{1}{1 - \lambda_S} + \mu\frac{\lambda_A}{1 - \lambda_S}U'(W_2 - g(e_2) - \lambda_A(S_2 - W_2)) - \frac{\gamma\lambda_A}{p_2(1 - \lambda_S)} + \frac{\pi(1 + \lambda_A - \lambda_S)}{p_2(1 - \lambda_S)} \tag{2}$$

$$\nu V'(S_3-\lambda_S(S_3-W_3)) = \frac{1}{1-\lambda_S}+\mu\frac{\lambda_A}{1-\lambda_S}U'(W_3-g(e_3)-\lambda_A(S_3-W_3))+\frac{\gamma\lambda_A}{p_3(1-\lambda_S)}+\frac{(\psi-\pi)(1+\lambda_A-\lambda_S)}{p_3(1-\lambda_S)} \tag{3}$$

$$\nu V'(S_4-\lambda_S(S_4-W_4)) = \frac{1}{1-\lambda_S}+\mu\frac{\lambda_A}{1-\lambda_S}U'(W_4-g(e_4)-\lambda_A(S_4-W_4))-\frac{\psi(1+\lambda_A-\lambda_S)}{p_4(1-\lambda_S)} \tag{4}$$

$$\mu U'(W_1-g(e_1)-\lambda_A(S_1-W_1)) = \frac{1}{1+\lambda_A}-\nu\frac{\lambda_S}{1+\lambda_A}V'(S_1-\lambda_S(S_1-W_1)) \tag{5}$$

$$\mu U'(W_2-g(e_2)-\lambda_A(S_2-W_2)) = \frac{1}{1+\lambda_A}-\nu\frac{\lambda_S}{1+\lambda_A}V'(S_2-\lambda_S(S_2-W_2))+\frac{\gamma}{p_2}+\frac{\pi(1+\lambda_A-\lambda_S)}{p_2(1+\lambda_A)} \tag{6}$$

$$\mu U'(W_3-g(e_3)-\lambda_A(S_3-W_3)) = \frac{1}{1+\lambda_A}-\nu\frac{\lambda_S}{1+\lambda_A}V'(S_3-\lambda_S(S_3-W_3))-\frac{\gamma}{p_3}+\frac{(\psi-\pi)(1+\lambda_A-\lambda_S)}{p_3(1+\lambda_A)} \tag{7}$$

$$\mu U'(W_4-g(e_4)-\lambda_A(S_4-W_4)) = \frac{1}{1+\lambda_A}-\nu\frac{\lambda_S}{1+\lambda_A}V'(S_4-\lambda_S(S_4-W_4))-\frac{\psi(1+\lambda_A-\lambda_S)}{p_4(1+\lambda_A)} \tag{8}$$

$$\mu U'(W_1-g(e_1)-\lambda_A(S_1-W_1))g'(e_1) = 1 \tag{9}$$

$$\mu U'(W_2-g(e_2)-\lambda_A(S_2-W_2))g'(e_2)-\frac{(\gamma+\pi(1-2\lambda_S))}{p_2}g'(e_2-\Delta\theta) = 1 \tag{10}$$

$$\mu U'(W_3-g(e_3)-\lambda_A(S_3-W_3))g'(e_3)+\frac{\gamma+(\pi-\psi)(1-2\lambda_S)}{p_3}g'(e_3) = 1 \tag{11}$$

$$\mu U'(W_4-g(e_4)-\lambda_A(S_4-W_4))g'(e_4)+\frac{\psi(1-2\lambda_S)}{p_4}g'(e_4) = 1. \tag{12}$$

Proof of Proposition 1: Substituting (5), (6), (7), (8) into (9), (10), (11), (12) gives that $g'(e_1) = g'(e_3) = g'(e_4) = \frac{1+\lambda_A-\lambda_S}{1-2\lambda_S}$ and $g'(e_2) < \frac{1+\lambda_A-\lambda_S}{1-2\lambda_S}$. Since $g''(e_i) > 0$, the rank of effort levels is $e_1 = e_3 = e_4 > e_2$. Suppose $\lambda_A = -\lambda_S$. Then $g'(e_1) = g'(e_3) = g'(e_4) = 1$ and $g'(e_2) < 1$. This implies that $e_1 = e_3 = e_4 = e^* > e_2$.

Suppose $\lambda_A < -\lambda_S$. Then $g'(e_2) < g'(e_1) = g'(e_3) = g'(e_4) < 1$. This means that $e^* > e_1 = e_3 = e_4 > e_2$. Upper boundary of $g'(e_2)$ goes to $\frac{1+\lambda_A-\lambda_S}{1-2\lambda_S}$ in our case. Thus, the principal sets $g'(e_2) = \frac{1+\lambda_A-\lambda_S}{1-2\lambda_S} - \varepsilon$ where $\varepsilon > 0$, in order to guarantee the maximum output level. Since $\frac{1+\lambda_A-\lambda_S}{1-2\lambda_S} - \varepsilon = g'(e_2) < g'(e_2^B) = 1-\varepsilon$, the ranking of effort levels is $e_2 < e_2^B$.

Now suppose $\lambda_A > -\lambda_S$. Then $g'(e_1) = g'(e_3) = g'(e_4) > 1$. This implies that $e_1 = e_3 = e_4 = e^*$. Upper boundary of $g'(e_2)$ also increases to $\frac{1+\lambda_A-\lambda_S}{1-2\lambda_S}$ in this case. Thus, the principal sets $g'(e_2) = \frac{1+\lambda_A-\lambda_S}{1-2\lambda_S} - \varepsilon$ where $\varepsilon > 0$, in order to get the maximum profit. Since $g'(e_2) > g'(e_2^B) = 1-\varepsilon$, we have $e_2 > e_2^B$. For a given ε (where $\frac{\partial\varepsilon}{\partial\Delta\theta} > 0$), when we have $\frac{\lambda_A+\lambda_S}{1-2\lambda_S} = \varepsilon$ we get $g'(e_2) = g'(e^*) = 1$. Therefore, there exist some values for λ_A and λ_S where $e_2 = e^*$. Finally, it is easy to see that $e_2 > e^*$ when $\frac{\lambda_A+\lambda_S}{1-2\lambda_S} > \varepsilon$, and $e_2 < e^*$ when $\frac{\lambda_A+\lambda_S}{1-2\lambda_S} < \varepsilon$.

Q.E.D.

Proof of Proposition 2: First substitute (5), (6), (7) and (8) in (1), (2), (3) and (4) to get the following equations:

$$\nu V'(S_1 - \lambda_S(S_1 - W_1)) = \frac{1+2\lambda_A}{1+\lambda_A-\lambda_S} \tag{13}$$

$$\nu V'(S_2 - \lambda_S(S_2 - W_2)) = \frac{1+2\lambda_A}{1+\lambda_A-\lambda_S} + \frac{\pi(1+2\lambda_A)}{p_2} \tag{14}$$

$$\nu V'(S_3 - \lambda_S(S_3 - W_3)) = \frac{1+2\lambda_A}{1+\lambda_A-\lambda_S} + \frac{(\psi-\pi)(1+2\lambda_A)}{p_3} \tag{15}$$

$$\nu V'(S_4 - \lambda_S(S_4 - W_4)) = \frac{1+2\lambda_A}{1+\lambda_A-\lambda_S} - \frac{\psi(1+2\lambda_A)}{p_4}. \tag{16}$$

Now, we use (1), (2), (3) and (4) in (5), (6), (7) and (8) to get the following equations:

$$\mu U'(W_1 - g(e_1) - \lambda_A(S_1 - W_1)) = \frac{1-2\lambda_S}{1+\lambda_A-\lambda_S} \tag{17}$$

$$\mu U'(W_2 - g(e_2) - \lambda_A(S_2 - W_2)) = \frac{1-2\lambda_S}{1+\lambda_A-\lambda_S} + \frac{\gamma}{p_2} + \frac{\pi(1-2\lambda_S)}{p_2} \tag{18}$$

$$\mu U'(W_3 - g(e_3) - \lambda_A(S_3 - W_3)) = \frac{1-2\lambda_S}{1+\lambda_A-\lambda_S} - \frac{\gamma}{p_3} + \frac{(\psi-\pi)(1-2\lambda_S)}{p_3} \tag{19}$$

$$\mu U'(W_4 - g(e_4) - \lambda_A(S_4 - W_4)) = \frac{1-2\lambda_S}{1+\lambda_A-\lambda_S} - \frac{\psi(1-2\lambda_S)}{p_4}. \tag{20}$$

To show that (AIC) is binding, suppose $\gamma = 0$. Then, using the conditions (14), (15) and (18), (19), we get the following equality

$$\frac{V'(S_2 - \lambda_S(S_2 - W_2))}{V'(S_3 - \lambda_S(S_3 - W_3))} = \frac{U'(W_2 - g(e_2) - \lambda_A(S_2 - W_2))}{U'(W_3 - g(e_3) - \lambda_A(S_3 - W_3))}. \tag{21}$$

On the other hand, (AIC) implies that

$$W_3 - g(e_3) - \lambda_A(S_3 - W_3) \geq W_2 - g(e_2 - \Delta\theta) - \lambda_A(S_2 - W_2) > W_2 - g(e_2) - \lambda_A(S_2 - W_2). \tag{22}$$

From (21) and (22), we get the following inequality

$$S_3 - \lambda_S(S_3 - W_3) > S_2 - \lambda_S(S_2 - W_2). \tag{23}$$

The equations (22) and (23) mean that $S_3 - \lambda_S(S_3 - W_3) + W_3 - g(e_3) - \lambda_A(S_3 - W_3) > S_2 - \lambda_S(S_2 - W_2) + W_2 - g(e_2 - \Delta\theta) - \lambda_A(S_2 - W_2)$, i.e. $(CIC3)$ does not bind. This implies that $\pi = 0$. Then, the equations (18) and (19) imply that:

$$W_2 - g(e_2) - \lambda_A(S_2 - W_2) \geq W_3 - g(e_3) - \lambda_A(S_3 - W_3). \tag{24}$$

Note that (22) and (24) cannot hold at the same time. This implies that there is a contradiction which completes this part of our proof and shows that $\gamma > 0$, i.e. (AIC) is binding.

Next, suppose that the second collusion constraint is not binding, i.e. $\psi = 0$. The equations (19) and (20) imply that $W_3 - g(e_3) - \lambda_A(S_3 - W_3) > W_4 - g(e_4) - \lambda_A(S_4 - W_4)$. Using $\lambda_A > 0$ we get

$$\frac{W_3 - g(e_3) - \lambda_A(S_3 - W_3)}{1 + 2\lambda_A} > \frac{W_4 - g(e_4) - \lambda_A(S_4 - W_4)}{1 + 2\lambda_A}. \tag{25}$$

From (15) and (16), we also have $S_3 - \lambda_S(S_3 - W_3) \geq S_4 - \lambda_S(S_4 - W_4)$. Using the assumption that $\lambda_S < 0.5$ we can write

$$\frac{S_3 - \lambda_S(S_3 - W_3)}{1 - 2\lambda_S} \geq \frac{S_4 - \lambda_S(S_4 - W_4)}{1 - 2\lambda_S}. \tag{26}$$

The equations (25) and (26) imply that

$$\frac{S_3 - \lambda_S(S_3 - W_3)}{1 - 2\lambda_S} + \frac{W_3 - g(e_3) - \lambda_A(S_3 - W_3)}{1 + 2\lambda_A} > \frac{S_4 - \lambda_S(S_4 - W_4)}{1 - 2\lambda_S} + \frac{W_4 - g(e_4) - \lambda_A(S_4 - W_4)}{1 + 2\lambda_A}, \tag{27}$$

which violates $(CIC2)$. Thus, $\psi > 0$ and $(CIC2)$ is binding.

We now show that $(CIC3)$ is binding. Assume to the contrary that $(CIC3)$ is not binding, i.e. $\pi = 0$. Then, the equations (14) and (15) imply that $S_2 - \lambda_S(S_2 - W_2) > S_3 - \lambda_S(S_3 - W_3)$.

We know that (AIC) is binding, so $(CIC3)$ can now be stated as $(CIC3') : S_3 - \lambda_S(S_3 - W_3) \geq S_2 - \lambda_S(S_2 - W_2)$. However, this is a contradiction to the implication of equations (14) and (15). Therefore, $\pi > 0$ and $(CIC3)$ is binding.

With the following proof of Proposition 3, we show that $(CIC1)$ is already satisfied with the current solution and hence $(CIC1)$ is not binding.

Q.E.D.

Proof of Proposition 3: We know that both (AIC) and $(CIC3)$ are binding. This fact implies that $S_2 - \lambda_S(S_2 - W_2) = S_3 - \lambda_S(S_3 - W_3)$. Moreover, (13), (14) and (16) imply that $S_4 - \lambda_S(S_4 - W_4) > S_1 - \lambda_S(S_1 - W_1) > S_2 - \lambda_S(S_2 - W_2)$. Therefore, the ranking of the supervisor's utilities at different states is $S_4 - \lambda_S(S_4 - W_4) > S_1 - \lambda_S(S_1 - W_1) > S_2 - \lambda_S(S_2 - W_2) = S_3 - \lambda_S(S_3 - W_3)$. From equations (17), (18) and (20), we have

$$W_4 - g(e_4) - \lambda_A(S_4 - W_4) > W_1 - g(e_1) - \lambda_A(S_1 - W_1) > W_2 - g(e_2) - \lambda_A(S_2 - W_2). \tag{28}$$

Combining the fact that $(CIC2)$ is binding and $g(e_3) = g(e_4)$ yields that

$$\frac{S_4 - \lambda_S(S_4 - W_4)}{1 - 2\lambda_S} + \frac{W_4 - \lambda_A(S_4 - W_4)}{1 + 2\lambda_A} = \frac{S_3 - \lambda_S(S_3 - W_3)}{1 - 2\lambda_S} + \frac{W_3 - \lambda_A(S_3 - W_3)}{1 + 2\lambda_A}.$$

Since $S_4 - \lambda_S(S_4 - W_4) > S_3 - \lambda_S(S_3 - W_3)$, we have $W_3 - g(e_3) - \lambda_A(S_3 - W_3) > W_4 - g(e_4) - \lambda_A(S_4 - W_4)$. This completes the proof of the following ranking $W_3 - g(e_3) - \lambda_A(S_3 - W_3) > W_4 - g(e_4) - \lambda_A(S_4 - W_4) > W_1 - g(e_1) - \lambda_A(S_1 - W_1) > W_2 - g(e_2) - \lambda_A(S_2 - W_2)$. Now, it can be easily verified that $(CIC1)$ is already satisfied and not binding, since $S_1 - \lambda_S(S_1 - W_1) > S_2 - \lambda_S(S_2 - W_2)$ and $W_1 - g(e_1) - \lambda_A(S_1 - W_1) > W_2 - g(e_2) - \lambda_A(S_2 - W_2)$.

Q.E.D.