Coordination with Indirect Messages*

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Abstract

We theoretically investigate the effect of allowing one-sided communication with costless indirect messages on stag-hunt game outcomes. Since Heinemann et al. (2009) show that players who avoid risk also avoid strategic uncertainty, we chose a sender's level of risk aversion as the indirect message. We show that if both sender and receiver interpret the message content similarly, it is possible that they can end up either on the risk-dominant or on the payoff-dominant equilibrium. We also show that players in the extreme risk groups are willing to declare risk attitudes truthfully to increase the probability of coordination. On the other hand, players in the medium risk-averse group are willing to mimic the risk loving group to achieve efficient coordination.

Keywords: stag-hunt game, coordination, risk information, costless messages

JEL classification codes: C72, D82

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1 Introduction

We analyze the outcome of a stag-hunt game with pre-play one way costless communication. One of the players, sender, is given the chance to signal her risk attitude to the other player, receiver, before playing the game. The signal about the player's risk attitude is indirect and requires interpretation. Such an indirect signal is important for the game when the payoffs from the game are monetary and strategic risk exists in the game. In a stag-hunt game (Table 1), players choose between strategically safe ("A") and risky ("B") actions. The possible outcomes for this game are: the payoff-dominant equilibrium (B, B), the risk-dominant equilibrium (A, A), and out of equilibrium: (A, B) and (B, A).

Table 1: 2x2 Stag-Hunt Game

	A	B
A	$570, 570$	$570, 70$
B	$70, 570$	$770, 770$

An indirect signal about a player's risk attitude may affect a player's action (hence the game outcome) for the following reasons: First, there is a relationship between risk and strategic uncertainty. According to Knight (1921) there are two kinds of uncertainty: exogenous uncertainty or risk with given a priori probabilities for all possible states of the world (i.e., lotteries), and endogenous uncertainty given by the lack of such probabilities. Heinemann et al. (2009) found that players who avoid risk also avoid strategic uncertainty.[1] Since there is a strategic risk in the game shown in Table 1, giving a player a chance to send a message about her risk attitude may be beneficial for achieving efficient coordination.

Second, agents' utility representations differ according to their risk aversion. In the game, risk-averse, risk-neutral, and risk-loving agents expect different payoffs from playing "A" (or "B") for the same belief about the other person's action choice. Hence, their optimal action choices may change as a response to their beliefs: It may be optimal for a risk-averse agent to play "A" even when she thinks that her opponent plays "A" with a low probability. On

[1]Strategic uncertainty is defined in Heinemann et al. (2009) as uncertainty concerning purposeful behavior of players in an interactive decision situation. See also Bohnet and Zeckhauser (2004), Schechter (2007) and Lange et al. (2011) for more on the relationship between risk-aversion and uncertainty in strategic games.

the other hand, it may be optimal for a risk-loving agent to play "A" only when she thinks that her opponent plays "A" with a high probability. This difference stems from the concave (convex) utility function of a risk-averse (risk-loving) agent. Similarly, when a player gets a signal about how risk averse the other person is, she may form her beliefs accordingly. She may expect a risk-averse (risk-loving) opponent to play "A" with a higher (lower) probability and best respond to her belief by playing "A" ("B").

In this paper, we characterize a perfect Bayesian equilibrium in which agents can use such a message stage to achieve coordination. In the model, we first assume that players belong to one of three groups according to their risk aversion: Group 1 is the risk-loving group, Group 2 is the medium risk-averse group, and Group 3 is the most risk-averse group. Given a player knows her own group but only the risk aversion distribution of the other player, one of the players, sender, has a chance to send a message to her opponent. This message does not have to be truthful. We define a system of beliefs such that the receiver always believes that the message is true. [2] After the costless message stage, agents choose their strategies for the game in Table 1. We show that under these conditions, in a Perfect Bayesian equilibrium, a sender, who is in Group 1 or Group 2, sends Group 1 message, and she plays the risky action, "B", afterwards. A Group 3 sender sends Group 3 message and plays the safe action, "A", afterwards. A receiver, who receives Group 1 (Group 3) message, plays "B" ("A"). This implies that a sender in extreme risk groups (Group 1 and Group 3) is willing to declare her risk attitude truthfully to increase the probability of coordination. On the other hand, a sender in the medium risk-averse group (Group 2) is willing to mimic the risk loving group to achieve efficient coordination.

2 Analysis

We now provide a model to show that coordination can be achieved with the possibility of indirect communication. We assume that there are two players: a sender (S) and a receiver

[2]There can be many outcomes in this game, i.e., players can end up with coordination failure. By assuming that the receiver believes the message is truthful and the sender sends a message strategically to increase their coordination in either equilibrium, the coordination problem in an equilibrium can be overcome.

(R). We label the sender as player 1 and the receiver as player 2. Player i has a risk aversion parameter, r_i, belonging to one of 10 different risk categories, where each category consists of continuously divided intervals in $\mathbb{R}$.[3] If r_i is in risk categories $1, 2, 3$ or 4, we say that player i is in Group 1. If r_i is in risk categories $5, 6$ or 7, we say that player i is in Group 2. Moreover, r_i is in risk categories $8, 9$ or 10 implies that player i is in Group 3. We say that player $i \in$ risk category k if $r_i \in [L_k, U_k)$ where $\cup_{k=1}^{k=10} [L_k, U_k) = \mathbb{R}$. For each risk category, we define a representative risk aversion parameter which is equal to the average of the upper and lower bound for that category.[4] Each player only knows her own risk aversion parameter and the probability distribution of the other player's risk aversion. Due to observed data in risk elicitation experiments using the Holt and Laury method[5], we assume that all individual risk aversion parameters are normally distributed with a mean of 0.28 and a standard deviation of 0.25.

The sender has an option to send a message m from the set of feasible messages M. Let $M = \{\text{Group 1, Group 2, Group 3}\}$ be the set of feasible messages and $\mathcal{M} = M \cup \emptyset$. An empty message, $\emptyset$, represents the no message option for the sender. The receiver observes the message coming from the sender. With this knowledge, each agent simultaneously chooses the safe or the risky action for the game in which monetary payoffs in Table 1 are converted into CRRA utilities where the utility of a player i is $u_i(x) = \frac{x^{1-r_i}}{1-r_i}$, in which x is a monetary payment and r_i is the risk aversion coefficient.

[3]These 10 intervals are similar to the intervals in Holt and Laury (2002).

[4]These representative risk aversion parameters can be seen in Table 2. For the first and last risk categories we take 1 below the upper bound and 1 above the lower bound respectively.

[5]We determine the distribution according to observed proportions in low-real treatment of Holt and Laury (2002).

Table 2: Risk Categories and Representative Risk Aversion Parameter

Risk Category k	Group	$[L_k, U_k)$	Representative Risk Aversion Parameter
1	1	$[-\infty, -0.95)$	-2
2	1	$[-0.95, -0.49)$	-0.72
3	1	$[-0.49, -0.15)$	-0.32
4	1	$[-0.15, 0.15)$	0
5	2	$[0.15, 0.41)$	0.28
6	2	$[0.41, 0.68)$	0.55
7	2	$[0.68, 0.97)$	0.83
8	3	$[0.97, 1.37)$	1.17
9	3	$[1.37, 1.53)$	1.45
10	3	$[1.53, \infty)$	2.5

2.1 Results

The game is symmetric for the agents who belong to the same risk category but there is an asymmetry in utilities if the agents do not belong to the same risk category. A pure strategy perfect Bayesian equilibrium is a system of beliefs[6] and profile of strategies (m^*, s_1^*, s_2^*) which are best responses to each other while maximizing the following conditional expected payoff for each player's each possible risk aversion coefficient after the communication stage:

$$s_i \in argmax_{\{\mathrm{A,\ B}\}} \mathbb{E}_{r_{-i}} U_i(s_i, s_{-i}^*(r_{-i}), r_i | m^* \in \mathcal{M}). \tag{1}$$

Let us denote the equilibrium probability that player 2 plays "A" after a message $m \in \mathcal{M}$ given player 2's type is r_2 by p_2. Given this belief

$$\mathbb{E}_{r_2} U_1(s_1 = \mathrm{A}, s_2^*(r_2), r_1 | m \in \mathcal{M}) = \frac{570^{1-r_1}}{1-r_1} \tag{2}$$

and

$$\mathbb{E}_{r_2} U_1(s_1 = \mathrm{B}, s_2^*(r_2), r_1 | m \in \mathcal{M}) = p_2 \frac{70^{1-r_1}}{1-r_1} + (1-p_2) \frac{770^{1-r_1}}{1-r_1}. \tag{3}$$

[6]See Mas-Colell et al. (1995) for a formal definition of system of beliefs in dynamic games.

So, playing "A" is optimal for player 1 if:

$$570^{1-r_1} > p_2 70^{1-r_1} + (1-p_2)770^{1-r_1}. \tag{4}$$

In other words, playing "A" is optimal for player 1 if $p_2 > \frac{770^{1-r_1}-570^{1-r_1}}{770^{1-r_1}-70^{1-r_1}} := F(r_1)$ or taking the inverse of the function F if $r_1 > F^{-1}(p_2)$. By symmetry of monetary payoffs player 2 will have a similar equilibrium strategy, i.e., "A" is optimal for player 2 if $p_1 > \frac{770^{1-r_2}-570^{1-r_2}}{770^{1-r_2}-70^{1-r_2}} = F(r_2)$ or $r_2 > F^{-1}(p_1)$. Note that $F(r) = \frac{770^{1-r}-570^{1-r}}{770^{1-r}-70^{1-r}}$ is decreasing in r.

The above arguments imply that the equilibrium follows a certain pattern: Player i from risk category k plays "B" if $r_i \in (-\infty, r_k^*)$ and plays "A" if $r_i \in [r_k^*, \infty)$.[7] Given such an equilibrium strategy, the probability that the other player, player $-i$, plays "A" is $p_{-i} = P(r_{-i} > r_k^*)$. For a category representative risk parameter, one can find bounds for p_{-i}. Since it is assumed that the distribution of risk parameters is $N(0.28, 0, 25)$, we can also find the cut off values, r_k^*, for each risk category k. If player i belongs to the first risk category, it is optimal for her to play "A" if her belief about her opponent's safe action choice is higher than $0.59 = F(-2)$.[8] This implies that player i in the first risk category believes that her opponent chooses "A" if $r_{-i} > r_1^*$, chooses "B" if $r_{-i} < r_1^*$. The highest belief that leads a player in the first category to choose "B" is 0.58, i.e., $Prob(r_{-i} < r_1^* | r_i = -2) = 0.58$. Given that risk aversion distribution is $N\,(0.28, 0.25)$, r_1^* can be found as 0.33. Given the equilibrium beliefs in each risk category, the cut off risk aversion parameters r_2^*, r_3^*, r_4^*, r_5^*, r_6^*, r_7^*, r_8^*, r_9^* and r_{10}^*, can be found as 0.22, 0.18, 0.14, 0.1, 0.06, 0.02, -0.04, -0.09, and -0.23 respectively.[9]

By using Table 2 and these cut off parameters, player i from risk category 1 ($r_1^* = 0.33$) plays "B" if $r_{-i} \in (-\infty, L_6)$ and plays "A" if $r_{-i} \in [L_6, \infty)$. That is, player i plays the risky action if she thinks that the other player belongs to the first five risk categories, and plays the safe action if she thinks that the other player belongs to the last five risk categories.

[7] We assume without loss of generality that player i plays "A" whenever she is indifferent.

[8] The critical value is found using the representative risk aversion parameter for the 1^{st} risk category in Table 2. The critical beliefs for the 2^{nd}, 3^{rd}, 4^{th}, 5^{th}, 6^{th},7^{th}, 8^{th}, 9^{th} and 10^{th} risk categories are 0.41, 0.34, 0.29, 0.24, 0.19, 0.15, 0.1, 0.07, and 0.02 respectively.

[9] If the cut off we found for the risk category is above (below) the representative risk aversion parameter for that category, we assume the agent plays "B" ("A") for that category.

Thefore, the critical level of opponent's risk category to switch from playing the risky action to playing the safe action is risk category 6 for the player. Similarly, the critical level of opponent's risk category is 5 for a player belonging to risk categories $2 \leq k \leq 7$, and it is risk category 4 for a player belonging to risk categories $8 \leq k \leq 10$.

Given these optimal strategies for the last stage of the game, we can find the optimal messages for the communication stage. Suppose that Group 1 message is received and the receiver believes that this message is truthful. That is, the sender's risk category is either $1, 2, 3$, or 4. Then, it is optimal for a receiver belonging to risk categories $1 \leq k \leq 7$ to play "B" by the definition of the equilibrium strategy. Note that the the critical level of sender's risk category 4 for a receiver belonging to risk categories $8 \leq k \leq 10$. The expected utility of playing "A" for player 2 (receiver) in risk category 8 ($r_2 = 1.17$) is $\mathbb{E}U_2(s_1^*(r_1), s_2 = \text{A}, r_2|m = \text{Group 1}) = \frac{570^{1-r_2}}{1-r_2}$ and the expected utility of playing "B" is $\mathbb{E}U_2(s_1^*(r_1), s_2 = \text{B}, r_2|m = \text{Group 1}) = \mathit{prob}(r_1 \leq -0.32|m = \text{Group 1})\frac{770^{1-r_2}}{1-r_2} + \mathit{prob}(r_1 \in (-0.32, 0)|m = \text{Group 1})\frac{70^{1-r_2}}{1-r_2}$. Therefore, choosing "A" is optimal for player 2 in risk categories 8, 9, and 10 since $\mathbb{E}U_2(s_1^*(r_1), s_2 = \text{A}, r_2|m = \text{Group 1}) \geq \mathbb{E}U_2(s_1^*(r_1), s_2 = \text{B}, r_2|m = \text{Group 1})$. Thus, expected utility of player 1 (sender) by sending Group 1 message is $\mathbb{E}U_1(s_1 = \text{B}, s_2^*(r_2), r_1|m = \text{Group 1}) = \mathit{prob}(r_2 \leq 0.83)\frac{770^{1-r_1}}{1-r_1} + \mathit{prob}(r_2 \geq 0.83)\frac{70^{1-r_1}}{1-r_1}$. Note that $\mathbb{E}U_1(s_1 = \text{B}, s_2^*(r_2), r_1|m = \text{Group 1}) > \frac{570^{1-r_1}}{1-r_1}$ for a sender in Group 1 and Group 2. This implies that it is optimal for Group 1 and Group 2 senders to send Group 1 message and thus convey their intentions to play "B" (the risky action) rather than sending another message $m' \in \mathcal{M}\setminus\{\text{Group 1}\}$ and playing the safe action. However, Group 3 sender will not mimic other groups since $\mathbb{E}U_1(s_1 = \text{B}, s_2^*(r_2), r_1|m = \text{Group 1}) < \frac{570^{1-r_1}}{1-r_1}$ for all risk categories such that $k \geq 8$.[10] This implies that Group 3 sender sends Group 3 message to convey her intention to play "A", the safe action.[11] Therefore, pre-play indirect communication

[10]We assume that there is a pre-play communication whenever the sender is indifferent.

[11]A belief system is defined using Bayes' rule for the possible risk category nodes at the risk group information set on the equilibrium path. Moreover, off the equilibrium path beliefs are defined such that belief [1] is assigned to the highest risk category node, and belief [0] is assigned to the other nodes at the risk group information set. For example, given that information set Group 1 is on the equilibrium path, positive beliefs are assigned only to risk category nodes $1, 2, 3$, and 4 using the prior, and belief [0] is assigned to risk category nodes $5, 6, 7, 8, 9$, and 10. At the information set Group 2 (3) off the equilibrium path, belief [1] is

about risk attitudes can be used as a coordination device on both types of equilibrium. This completes the proof of the following result:

Proposition 1 *There exists a perfect Bayesian equilibrium in which a sender, who is in Group 1 or Group 2, sends Group 1 message and plays the risky action, "B", afterwards. A Group 3 sender sends Group 3 message and plays the safe action, "A", afterwards. A receiver, who receives Group 1 (Group 3) message, plays "B" ("A").*

3 Conclusion

In this paper, we present a stag hunt game in which agents are allowed to communicate with each other before playing the game through indirect messages. In this set up, we characterize a perfect Bayesian equilibrium in which agents use this type of messages to achieve coordination. We find that allowing players to use such indirect messages can be used as a coordination device on both equilibria. We also show that the extreme risk groups, risk loving group (Group 1) and the most risk-averse group (Group 3), are willing to declare risk attitudes truthfully to increase the probability of coordination. The medium risk-averse group are willing to mimic the risk loving group to achieve efficient coordination, and hence maximize the expected return. These results also provide a theoretical background for the experimental test of hypothesis 3 in Büyükboyacı and Küçükşenel (2017).

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assigned to the risk category node 7 (10) and belief [0] is assigned to the risk category nodes 5 and 6 (8 and 9).

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