Bargaining in legislatures over private and public goods with endogenous recognition

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Abstract

This paper studies a sequential model of multilateral bargaining under majority rule in which legislators make decisions in both private and public good dimensions via an endogenous recognition process. Legislators can expend resources to become the proposer and to make proposals about the allocation of private and public goods. We show that legislators exert unproductive effort to be the proposer and make proposals in both dimensions depending on legislative preferences. Effort choices in equilibrium depend mainly on preferences in both distributional and ideological dimensions as well as the patience level of legislators and the legislature's size. We also show that in a diverse legislature it may be possible to observe distributive policies when the majority of legislators have collective-leaning preferences, or vice versa.

Keywords: Legislatures and legislative processes, majority rule, public goods

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1 Introduction

Bargaining in legislatures and their internal dynamics are among the most significant topics discussed in both game theory and political economy. Particularly following Baron and Ferejohn's (1989) seminal contributions to this literature related to legislators' bargaining procedures over a fixed amount of collective surplus, many works have investigated the concepts of coalition formation, the structure of legislation, and voting strategies. One of the main findings of this literature is "proposer power". This means that the legislator who is selected as the proposer gets a larger share of the collective surplus than the other legislators do, and it turns out to be very important in modeling legislative decision-making (Baron 2019). The main focus of our work is to explain how such proposer power is gained by wastefully spending resources in a legislative game with a much broader policy space. In our policy space, legislators can allocate collective surplus to the policy dimensions of both private goods, from which only legislators who receive the surplus can benefit (e.g., pork barrel appropriations), and public goods, from which all legislators benefit (e.g., public roads). Moreover, we investigate how equilibrium rent-seeking and policy-making decisions change depending on the ideological positions (preferences between public and particularistic goods) of the legislators.

Another objective of this paper is to offer predictions about how legislators allocate resources between public and private goods under the constraint of a fixed budget. Owing to limited resources, a trade-off between those policy dimensions exists: increasing the public good apportionment benefits all legislators, but at the same time fewer resources become available for private goods, which may be necessary for a legislator's re-election (Mayhew 1974). Our model clarifies the conditions on ideological positions of legislators under which all resources are allocated to public good spending or private good spending, or both. We characterize legislators according to their valuations of private and public goods. One of our main findings is that if legislators assign more value to particularistic spending, the equilibrium provision of public good declines and wasteful rent-seeking efforts are increased to become the proposer in the policy-making process. In our model, we show that in a diverse legislature, it may be possible to have distributive policies when the majority has collective-leaning preferences, or vice versa. Moreover, the level of wasteful resource spending for attaining proposer power is positively correlated with the valuation of private good spending, and it is negatively correlated with the size of the legislature and legislators' level of patience.

In the relevant literature, some previous models examine legislative bargaining with only particularistic goods. Baron and Ferejohn (1989) present a private good bargaining model in legislatures, where legislators are recognized as the proposer randomly. On the contrary, in Rubinsteins (1982) model, agents are recognized as the proposer in a fixed sequence which is also consistent with Baron and Ferejohn (1989). Baron and Ferejohn's model examines coalition formation and allocation of private goods under closed and open rules. Eraslan (2002) shows that symmetric stationary equilibria are essentially unique in the Baron–Ferejohn model. Banks and Duggan (2000) generalize that model. They prove the existence of stationary equilibria when the set of alternatives is a compact and convex subset of multidimensional Euclidean space. Merlo and Wilson (1995) examine a "divide the dollar game" under unanimity rule with a stochastically changeable prize over time. In the work of Austen-Smith and Banks (1988), particularistic aspects are added to the spatial voting model and they identify equilibrium voting strategies, party positions, and legislative policy outcomes. On the other hand, some previous studies capture both particularistic and general policies. In those models, either it is hard to see the explicit trade-offs between collective and particularistic policies in the policy-making process or the recognition process is exogenous. Jackson and Moselle (2002) examine a legislative voting game in which both collective and particularistic policies are possible. However, they do not show the explicit trade-offs in the provision of public and private goods; the recognition process also is exogenous in their model. Volden and Wiseman (2007) present a sequential bargaining model in which legislators make decisions in both private and public good dimensions by a random recognition process.

Our model combines the models of Volden and Wiseman (2007) and of Yildirim (2007) and provides a unified framework for analyzing the legislative bargaining process over both private and public goods with endogenous recognition. Volden and Wiseman (2007) present results explaining the explicit trade-offs between private and public good spending. Moreover, they show that in a diverse legislature it becomes possible to see particularistic policies when the majority exhibits collective-leaning preferences, or vice versa. However, the model of Volden and Wiseman (2007) does not include the concept of rent-seeking.[1] We know that in a rent-seeking environment where bargaining occurs, agents generally exert effort or allocate resources to be recognized as the proposer. Such investments are made because being the proposer generally brings an extra surplus in sequential bargaining environments (see, e.g., Binmore and Eguia 2017). Yildirim (2007, 2010) studies a sequential bargaining model for particularistic goods wherein players exert effort to become the proposer and influence policy decisions. Yildirim (2007) also proves the existence of positive effort levels and shows the relationship among the players' marginal costs, patience, and effort levels with both transitory and persistent recognition under different voting rules. Unlike those models, our model shows how the rent-seeking behavior of legislators changes given that collective goods are also present in the policy-making process. Moreover, our model demonstrates that incorporating an endogenized costly recognition process into the model reduces total spending on the collective good. Furthermore, the existence of a collective spending dimension reduces the proposer's power and wasteful resource spending for recognition because of the non-excludable nature of public goods.

Our model also is related to the literature on dynamic legislative bargaining with endogenous status quo policies that include both private good and public good dimensions (Battaglini and Coate 2007; Bowen, Chen, and Eraslan 2014; Cho 2014). In those models,

[1]See Tullock (1980) for more on rent-seeking contests.

unlike our model, the game does not end when the proposal is accepted, and hence investment in the public good yields benefits for legislators in the future as well. In addition, the recognition process is either exogenous (Battaglini and Coate 2007; Bowen, Chen, and Eraslan 2014) or it can vary over time as a result of voters' electoral choices (Cho 2014). As far as we know, this is the first study of bargaining in legislatures over private and public goods with costly endogenous recognition, where the recognition process is endogenized using a rent-seeking contest. In our model, legislators expend resources to become the proposer and decide on how to allocate the collective surplus to private and public goods. Additionally, our bargaining game ends when the proposal on the floor is accepted. Otherwise, the game proceeds to the next session and similar steps are repeated.

The organization of the rest of the paper is as follows. Section 2 presents the model. Section 3 presents the benchmark cases for our main model. Section 4 analyzes the symmetric case. Section 5 relaxes the symmetry assumption and characterizes the equilibrium in a diverse legislature. Finally, Section 6 concludes the paper.

2 The model

2.1 Structure of the game

Legislators: Let $N = \{1, 2, ..., n\}$ be the set of legislators from different legislative districts who are making decisions on how to divide a fixed budget across constituencies. We assume that $|N| = n \geq 3$ and n is odd.

Decisions: A decision is a vector $(y, x_1, ..., x_n)$ consisting of an ideological decision, public good y, and a distributive decision, private good $(x_1, ..., x_n)$. The set of feasible decisions contains those such that $y \geq 0$ and $x_i \geq 0$ for all i and $y + \sum_{i \in N} x_i \leq 1$. Let D be the set of all feasible decisions: $D \equiv \{(y, x_1, ..., x_n) \mid \forall i \in N, x_i \geq 0, y \geq 0, y + \sum_{i \in N} x_i \leq 1\}$. Let d^i be the decision that legislator i proposes when he is recognized.

Recognition probabilities: Legislators simultaneously exert irreversible efforts in each legislative session. That is, their efforts can be renewed in each round of the legislature. Let e_i and $C_i(e_i)$ represent legislator i's effort and cost, respectively. For simplicity, we assume that $C_i(e_i) = k_i e_i$, where $k_i \in \mathbb{R}_+$ and $0 \leq e_i \leq \bar{e}$ for all $i \in N$.[2] Let $p_i(e) : [0, \bar{e}]^n \to [0, 1]$ be legislator i's recognition probability, such that

$$p_i(e) = \begin{cases} \frac{e_i}{\sum_{j \in N} e_j} & \text{if } \vec{e} \neq 0 \ ; \\ \frac{1}{n} & \text{if } \vec{e} = 0 \ . \end{cases}$$

Preferences: Legislators have preferences over decisions and effort levels. These preferences are represented by a utility function $u_i : \mathbb{R}^3_+ \to \mathbb{R}_+$. Utility function $u_i(y, x_i, e_i)$ is non-negative, continuous, strictly increasing in x_i, and strictly decreasing in e_i. We assume that legislator i's stage utility can be represented as

$$u_i(y, x_i, e_i) = \alpha_i x_i + (1 - \alpha_i) y - k_i e_i,$$

where $\alpha_i \in [0, 1]$ for all $i \in N$. We call α_i the ideological position (or the valuation of private good spending) of legislator i. Moreover, the discount rate of a legislator is denoted by δ, where $0 \leq \delta < 1$.

[2] Our model is equivalent to the model of legislative bargaining with exogenous recognition of Volden and Wiseman (2007) when $\bar{e} = 0$.

2.2 The legislative game

Let $T = \{t \in \mathbb{N} \mid t \leq t^*\}$ be a potentially infinite number of legislative sessions. At the beginning of the session $t = 0$, legislators exert effort simultaneously. Once efforts are chosen, a legislator is recognized with probability $p_i(e)$ to propose a decision in each session. Next, the recognized legislator proposes a decision $d^i = (y^i, x_1^i, ..., x_n^i)$. That proposal is then the motion on the floor. We assume that the amendment rule is closed, which implies that counter-proposals or amendments to the proposal are prohibited on the chamber's floor. Each legislator simultaneously decides whether to accept or reject the proposal. If a majority of legislators accept the proposal, the game ends and the decision is implemented. Otherwise, the game proceeds to the next session, legislators choose their effort levels again, and legislator j is recognized as the proposer with probability $p_j(e)$. If a decision $d \in D$ at session t is accepted, legislator i's payoff is given by $\delta^t(\alpha_i x_i + (1 - \alpha_i)y) - \sum_{t'=0}^{t} \delta^{t'} k_i e_i$. If no proposal has been approved by the end of the session, the default decision $(\bar{y}, \bar{x}_1, ..., \bar{x}_n)$ is implemented. We assume that $\bar{y} = \bar{x}_i = 0$ for all $i \in N$. Therefore, if no agreement is ever reached, legislature i's payoff is given by $-\sum_{t'=0}^{\infty} \delta^{t'} k_i e_i$. The homogeneous default decision (or status quo policy) assumption simplifies our analysis. If the default decision is not homogeneous, legislators with high outside options could be excluded from the minimal winning coalition since their compensation for accepting the proposal on the floor is higher than it is for the other legislators. Given that observation, legislators with high outside options may invest more in the recognition process and such a strategic move also may affect the other legislators' behavior. For more on legislative bargaining over particularistic goods with heterogeneous disagreement values, see Miller et al. (2018) and Kim and Kim (2019).

2.3 Strategies and legislative equilibrium

The game is one of perfect information and the definitions of strategies and sub-game perfection are standard. We also characterize the symmetric stationary equilibria for the game. A strategy is stationary if it is history-independent. An equilibrium is stationary if it is sub-game perfect and each legislator's strategy is stationary. A stationary equilibrium is symmetric if all legislators use the same strategy. The stationary sub-game perfect equilibrium (SSPE) can specify identical actions for each continuation of the game. Thus, by reducing the equilibrium set, solving the multiple equilibrium problem in multilateral bargaining becomes easier. As a result, since symmetric SSPE presents less complex and more tractable equilibria structures, we adopt stationary strategies and symmetric equilibrium. See Baron and Ferejohn (1989) and Jackson and Moselle (2002) for more on the equilibrium concept in legislative bargaining games.

3 Benchmarks

If $\alpha_i = 1$ for all $i \in N$ (implying that $y = 0$ in equilibrium), the model is equivalent to that of Yildirim (2007). If the recognition process is exogenous, our model has close connections with the models of Volden and Wiseman (2007), Jackson and Moselle (2002), and Baron and Ferejohn (1989). Suppose that the recognition process is exogenous. Our model is then equivalent to that of Volden and Wiseman (2007) if the recognition probabilities are uniform. If the ideological and distributive dimensions are not connected through the feasibility constraint on decisions, then our model is a special case of Jackson and Moselle (2002). Moreover, if decisions about public goods are not on the legislative agenda, the model is equivalent to that of Baron and Ferejohn (1989).

Benchmark 1: Baron and Ferejohn (1989): *If $\alpha_i = 1$ for all $i \in N$ and $\bar{e} = 0$, then* *in any stationary equilibrium each legislator's expected distributive allocation is* $\frac{1}{n}$*. Furthermore, there exists a stationary equilibrium in which any recognized legislator proposes a share* $(1-\delta\frac{(n-1)}{2n})$ *for himself and* $\frac{\delta}{n}$ *to each of* $\frac{n-1}{2}$ *randomly selected other legislators, and that allocation is approved by those randomly selected legislators. The first proposal receives a majority vote, so the legislature completes its task in the first session.*

The previous result captures the main idea of sequential bargaining. Legislators are offered part of the surplus, which makes them indifferent between voting "yes" now and waiting for the continuation. In the model, being a proposer carries a large advantage since the proposer keeps the excess surplus. Moreover, the recognition process is exogenous, something that is not related to any actual institution.

Benchmark 2: Yildirim (2007): *If* $\alpha_i = 1$ *for all* $i \in N$*, then under simple majority voting rule with* $\delta_i = \delta$ *and* $k_i = k$ *for all* $i \in N$*, there exists a unique equilibrium pair of* (e^*, v^*) *such that each* i *exerts the same amount effort with the same recognition probability.*

Benchmark 2 states that if agents are identical meaning that they have the same patience level with same marginal cost, then they have the same continuation values v^* with the same optimal effort level e^*. In the next section, we start to examine the symmetric case to present the intuition behind costly recognition and the interaction between the dimensions of public (or collective) good and private (or particularistic) good. In the symmetric case, all legislators have the same characteristics.

4 Symmetric case

In this section, we assume that legislators are identical ($\alpha_i = \alpha$ and $k_i = k$ for all $i \in N$) and we characterize equilibrium behavior depending on the legislators' valuations of private and public goods. The following propositions characterize the equilibrium for different values of

α. Our first result shows that if no conflict of interest exists in the legislature, legislators do not exert effort, which is assumed to be an unproductive activity, to increase the probability of recognition. Moreover, a recognized legislator contributes all surplus to the collective good given the legislators' ideological position. We call this the collective part of the equilibrium.

Proposition 1 *If $\alpha \in [0, \frac{1}{2})$, then a strategy profile is a symmetric stationary sub-game-perfect equilibrium if and only if it has the following form: Each legislator accepts any proposal from which he receives at least δ. The legislative game ends in the first session with a unanimously approved decision that involves only the public good dimension and thus a recognized legislator $k \in N$ makes an offer $d^k = (1, 0, ..., 0)$. Moreover, $e_i = e_j = 0$ and $p_i(e) = p_j(e) = \dfrac{1}{n}$ for all $i, j \in N$.*

The proof of Proposition 1 and subsequent results are relegated to the Appendix. Proposition 1 states that if legislators value the public good more highly relative to the private good, all available resources are devoted to the public good, which is non-excludable. This means that being recognized does not affect the expected payoffs. Hence, the proposer has no power. Therefore, legislators do not expend effort because such an action is costly and each legislator is equally likely to propose a decision in the equilibrium.

Proposition 2 i) *If $\frac{1}{2} \leq \alpha < \alpha_p^* < 1$, then a strategy profile is a symmetric stationary sub-game-perfect equilibrium if and only if the legislative game ends in the first session with a unanimously approved allocation that involves both public good and private good provision such that for all $j \in N$,*

- $e_j = e = \dfrac{(n-1)}{k}\left(\dfrac{\alpha(1-\alpha)(1-\delta)}{\delta\alpha + (1-\alpha)(1-\delta)n^2}\right) > 0.$
- $p_j(e) = \frac{1}{n}$. *A recognized legislator $i \in N$ makes an offer.*
- $y^i = y = \dfrac{\delta\alpha}{\delta\alpha + (1-\alpha)(1-\delta)n^2}.$
- $x_i^i = 1 - y = x = \dfrac{(1-\alpha)(1-\delta)n^2}{\delta\alpha + (1-\alpha)(1-\delta)n^2}$ *for himself and $x_{-i}^i = 0$ for other members of*

the legislature.

ii) *If $\alpha \geqslant \alpha_p^*$, then a strategy profile is a symmetric stationary sub-game-perfect equilibrium if and only if the legislative game ends in the first session with an approved decision that involves only private good provision such that for all $j \in N$,*

- $e_j = e' = \dfrac{\alpha(n-1)}{n^2 k}\left(1 - \dfrac{(n-1)\delta}{2n^2 - \delta(n-1)^2}\right) > 0.$
- $p_j(e) = \dfrac{1}{n}$. *A recognized legislator $i \in N$ makes an offer.*
- $y^i = 0, x_i^i = x' = \dfrac{2n^2 - \delta(n-1)n}{2n^2 - \delta(n-1)^2}$ *and* $\dfrac{2\delta}{2n^2 - \delta(n-1)^2}$ *to* $\frac{n-1}{2}$ *legislators selected at random.*[3]

The first part of Proposition 2 shows both collective and particularistic (mixed) dimensions of the equilibrium, and the second part of Proposition 2 shows the particularistic dimension of the equilibrium. In the mixed part of the equilibrium, if legislators expect to receive both private and public benefits from the bargaining process, they exert effort to increase their probability of recognition. Since all legislators expect to receive the same amount of private and public benefits, they exert the same level of effort. Thus, they are equally likely to make a proposal in the equilibrium. Moreover, as legislators assign more value to particularistic spending, the equilibrium provision of public good declines and it becomes zero after a certain point stated in the second part of Proposition 2. Notice that even though effort is costly, zero effort for recognition cannot be an equilibrium if the legislators expect to receive private benefits, unlike the collective part of the equilibrium.

We can now provide comparative statics results with respect to changes in the size of legislature, the patience level of legislators, and the ideological positions of legislators. We denote the equilibrium in the first part of Proposition 2 as the mixed part of the equilibrium.

[3]The cutoff point is $\alpha_p^* = \dfrac{n\left(1 - 2n - \delta n^2 + (\delta-1)n^3\right)}{\delta(n^4 - 2n^2 + 2n - 1) - n(n^3 + 2n^2 + 1)}$. Please see the proof of Proposition 2 for more on this.

Proposition 3 *In the mixed part of the equilibrium, i) the level of public good provision increases as legislators become more patient,* $\dfrac{\partial y}{\partial \delta} > 0$*; ii) the level of public good provision declines as the size of the legislature increases,* $\dfrac{\partial y}{\partial n} < 0$*; and iii) the level of public good provision increases as the valuation of private good spending increases,* $\dfrac{\partial y}{\partial \alpha} > 0$.

As legislators become more patient ($\delta \uparrow$), they become more willing to vote "no" on the current proposal, and they become more willing to wait on being the next proposer. To prevent that outcome, the current proposal must be made more attractive to the other members of the legislature. This goal only can be achieved through the public good spending channel in the mixed part of the equilibrium, implying that the level of public good provision increases as legislators become more patient. That is, $\dfrac{\partial y}{\partial \delta} > 0$. Note also that as the number of legislators or the size of the legislature falls, any coalition member's recognition probability increases in the next sessions. Therefore, a recognized legislator makes the proposal on the floor more attractive by spending more on the public good to prevent possible delays in the policy-making process. Thus, we have $\dfrac{\partial y}{\partial n} < 0$. The last relationship implies that public good spending increases in the mixed part of the equilibrium when legislators assign a higher value to private good spending. As α increases, the proposer has to offer more public good spending for the proposal to be accepted by a majority of legislators. Thus, in the mixed part of the equilibrium, the recognized legislator forms the winning coalition by increasing public good spending. Therefore, $\dfrac{\partial y}{\partial \alpha} > 0$.

We also can provide comparative statistics for optimal effort levels in the equilibrium.

Proposition 4 *In the mixed part of the equilibrium, i) the level of effort exerted by legislators declines as legislators become more patient,* $\dfrac{\partial e}{\partial \delta} < 0$*; and ii) the level of effort exerted by legislators declines as the marginal cost of effort increases,* $\dfrac{\partial e}{\partial k} < 0$*. In the mixed part of the equilibrium for sufficiently large legislatures,*[4]*, iii) the level of effort exerted by a legislator*

[4]If the number of legislators, n, is such that $n > \max\{\sqrt{\frac{\delta\alpha+(1-\delta)(1-\alpha)}{(1-\delta)(1-\alpha)}}+1, \sqrt{\frac{\delta}{(1-\delta)}\frac{\alpha}{(1-\alpha)}}\}$, we say that the legislature is sufficiently large. See the proof of Proposition 4 for more on this.

falls as the size of legislature grows, $\frac{\partial e}{\partial n} < 0$*; and iv) the level of effort exerted by a legislator increases as the value of private good spending increases,* $\frac{\partial e}{\partial \alpha} > 0$.

If legislators become more patient, δ increases, each legislator expects to receive larger shares of the total surplus. Thus, the recognized legislator invests more in the public good dimension to form the winning coalition in the mixed part of the equilibrium, implying that proposer power weakens owing to a reduction in expected private benefits. Therefore, each legislator exerts less effort in the recognition process and, hence, $\frac{\partial e}{\partial \delta} < 0$. Legislators exert less effort to be recognized as the proposer as the marginal cost of effort increases simply because of the direct cost-benefit analysis, $\frac{\partial e}{\partial k} < 0$. The third relationship for sufficiently large legislatures states that as the number of legislators increases, the equilibrium recognition probability declines directly and we thus observe less wasteful resource spending even though private benefits increase because of smaller allocations to the public good. Therefore, as n rises, the optimal effort level falls when the legislature is large. The last result is explained by the fact that as private good spending is valued highly (α increases), the proposer has to offer more public good spending to form a winning coalition in the mixed equilibrium. Thus, the proposer receives fewer private good benefits but more utility owing to increased α, resulting in an increase in proposer power when the legislature is large. Note also that the only way for the proposer to form the winning coalition is to offer more public good spending as α increases in the mixed part of the equilibrium, and the level of public good spending goes down when the legislature is sufficiently large. Therefore, wasteful resource spending in the proposer recognition process increases, $\frac{\partial e}{\partial \alpha} > 0$. When the legislature is small, the direction of the last two relationships changes. Note that more collective goods are provided in the equilibrium and the expected return to being the proposer is high when the legislature is small because of the larger probability of recognition. All else being equal, legislators are willing to increase their effort up to a certain threshold size of the legislature because of the direct cost-benefit analysis. If the valuation of private good spending increases when the legislature is small, the private benefit is limited owing to very large allocations to the public good, implying that a legislator receives a larger share of the surplus as a public good even though he is not the proposer. Therefore, proposer power weakens when the legislature is small given that both private and public good dimensions are present, leading to less wasteful effort being devoted to the recognition process as the values assigned to private good spending increase when the legislature is small.

We denote the equilibrium in the second part of Proposition 2 as the particularistic part of the equilibrium. The next proposition provides comparative statics for that part of the equilibrium.

Proposition 5 *In the particularistic part of the equilibrium, i) the level of effort exerted by legislators declines as legislators become more patient,* $\frac{\partial e^{'}}{\partial \delta} < 0$*; ii) the level of effort exerted by legislators declines as the marginal cost of effort increases,* $\frac{\partial e'}{\partial k} < 0$*; iii) the level of effort exerted by a legislator falls as the size of the legislature grows,* $\frac{\partial e^{'}}{\partial n} < 0$*; and iv) the level of effort exerted by a legislator increases as the valuation of private good spending increases* $(\alpha \uparrow)$*,* $\frac{\partial e^{'}}{\partial \alpha} > 0$*.*

The intuitions behind these comparative statics results are almost the same as those for the mixed part of the equilibrium in Proposition 4. The differences are related to the composition of benefits for the legislators and the results do not depend on the size of the legislature in the particularistic equilibrium. As legislators become more patient, or δ increases, each legislator expects to receive more private benefits, which also increases the probability of being excluded from the minimal winning coalition if not recognized as the proposer in the first session. Thus, in the particularistic part of the equilibrium, legislators do not invest more in the recognition process to attain proposer power since the cost of being excluded from the minimal winning coalition if not recognized outweighs the benefit of additional spending on being recognized, implying that we have $\frac{\partial e^{'}}{\partial \delta} < 0$. Moreover, legislators exert less effort in the recognition process as the marginal cost of effort increases simply because of the direct cost-benefit analysis. As the number of legislators increases, the equilibrium recognition probability declines and we thus observe less wasteful resource spending, $\frac{\partial e'}{\partial n} < 0$. The last relationship implies that as legislators assign more value to private good spending, they expect to receive larger shares of the total surplus. Wasteful rent-seeking efforts to become the proposer increase as α increases since proposer power increases. The power of the proposer increases because the proposer builds a minimal winning coalition by distributing the particularistic good so as to match the outside options of the coalition members and to keep the remaining large surplus to himself. Therefore, optimal effort level increases as α increases in the particularistic part of the equilibrium. Note that the sign of comparative statics results do not depend on the size of the legislature in the particularistic part of the equilibrium since no allocations to the public good are made in that type of equilibrium.

5 Extension to the asymmetric case

In this section, we relax the assumption that all legislators assign the same values to private good spending and assume that legislators have asymmetric α valuations. Without loss of generality, we concentrate on the simplest case of a diverse legislature.

5.1 Diverse legislature

We assume that the legislature is composed of two types of legislators. Let A denote the set of collective-leaning legislators (type 1) and B denote the set of particularistic-leaning legislators (type 2), where

$$A := \{i \in N | \ \alpha_i = \alpha_c = 0\} \text{ and } B := \{j \in N | \ \alpha_j = \alpha_p = 1\}.$$

Set A denotes the legislators who want to allocate the whole surplus to the collective good (public good dimension) and set B denotes the legislators who want to allocate the whole surplus to the private good (particularistic dimension). Note that $N = A \cup B, A \cap B = \varnothing$, and the legislature comprises two opposing political groups. All legislators have the same characteristics except for valuations α on the private and public good dimensions. Our model considers the most extreme case in which two types of legislators have completely different valuations. We show that both collective and distributive decisions exist in the equilibrium under some conditions. Therefore, it can be inferred that when we relax the assumption about the general structure of the legislature, it will still be possible to see both collective and distributive decisions in the equilibrium.

Let $|N|= n$ and $|A|= m$. Suppose that any $i \in A$ exerts effort e_i and any $j \in B$ exerts effort e_j, where $e_i, e_j \in \mathbb{R}_+$ for all $i, j \in A \cup B$.[5] Thus, the recognition probabilities for two types of legislators can respectively be defined as

$$p_i(e) = \frac{e_i}{me_i + (n-m)e_j} \text{ and } p_j(e) = \frac{e_j}{me_i + (n-m)e_j}.$$

Note that legislators exerting more effort to become the proposer expect to receive a larger share of the total surplus. Thus, the recognized legislator does not include the legislators exerting high levels of effort in the minimum winning coalition because of their higher cost levels and, hence, requirements of more generous compensation for accepting the proposal. We first show that each legislator exerts positive effort to have a chance of attaining proposer

[5] Even though legislators have the same marginal costs, their effort levels differ due to legislators' valuations α on private and public good dimensions.

power in the equilibrium.

Proposition 6 *In a diverse legislature, each type of legislator exerts positive effort to raise their chances of being recognized as the proposer in the equilibrium regardless of the composition of the legislature.*

Proposition 6 states that each legislator has an incentive to exert positive effort. That result is reasonable since a conflict of interest exists between the two types of legislators and being a proposer increases the chances of designing a better proposal for himself. Without exerting strictly positive effort, legislators' stage utility levels will be strictly less than their expected utilities.

In the next propositions, we characterize the equilibrium proposals that specify the allocation of private and public goods in the diverse legislature.

Proposition 7 *If the majority is composed of collective-leaning legislators ($m \geqslant \frac{n+1}{2}$) and a collective-leaning legislator $i \in A$ is recognized, then legislator i proposes no private good to any legislators and allocates the entire surplus to the public good dimension. If a particularistic-leaning legislator $j \in B$ is recognized, then legislator j proposes*

$$x_j^j = \frac{(1-\delta+\delta k e_i^*)(m e_i^* + (n-m)e_j^*)}{m e_i^* + (1-\delta)(n-m)e_j^*}$$

as private good for himself and gives no private good to the other legislators. Furthermore, legislator j offers

$$y^j = \frac{\delta e_i^*(m(1-k e_i^*) - k e_j^*(n-m))}{m e_i^* + (1-\delta)(n-m)e_j^*}$$

as public good.

Note that decision d is implemented in the first session. Moreover, owing to its complicated structure, we do not state the explicit form of the equilibrium level of efforts for each type, e_i^* and e_j^*, in the diverse legislature. It is easy to see that e_i^* for all $i \in A$ and e_j^* for all

$i \in B$ are the unique effort levels, which can be found using classical fixed-point arguments. Next, we provide a numerical example to present the relationship among optimal allocations, effort levels, size of the legislature, and legislators' patience levels.

Example 1 Suppose that the legislature comprises two collective-leaning (Type I) legislators and one particularistic-leaning (Type II) legislator. It is clear that if any collective-leaning legislator is recognized as the proposer, then the game ends immediately with the equilibrium decisions $x^* = 0$ and $y^* = 1$. Table 1 depicts the equilibrium decisions and effort levels depending on the legislators' patience level (δ) when the particularistic-leaning legislator is recognized. Note that the marginal cost of effort is fixed at k=1 for all legislators.

Table 1: Equilibrium effort levels and allocation decisions with two collective-leaning legislators and one particularistic-leaning legislator

	k=1	k=1	k=1
	$\delta = 0.1$	$\delta = 0.4$	$\delta = 0.6$
e_i^*	0.214404	0.184262	0.154617
e_j^*	0.221976	0.216249	0.202817
$y*$	0.04602	0.209341	0.353615
x^*	0.95398	0.790659	0.646385

As legislators become more patient, the equilibrium effort levels exerted by the legislators become smaller. That is because legislators, as they become more patient, are willing to reject a proposal on the floor and get a chance to become the proposer in the next session if the current proposal in not favorable. Proposer power weakens and hence both types of legislators invest less in the recognition process. In all three cases, we see that $e_i^* < e_j^*$, owing to the fact that the majority of legislators are collective-leaning. Since they form the majority, we expect to see that they exert less effort than the particularistic-leaning legislators. Nonetheless, that is not true in all possible cases. As we show in the next example, even though the majority consists of collective-leaning legislators, collective-leaning legislators exert more effort than particularistic-leaning ones unless their discount rates are not too high. That feature of the model is directly pertinent to the composition of the majority and legislators' patience levels. Besides, one of the most important outcomes of the first and the following example is that particularistic-leaning legislators may have a chance to capture almost the whole surplus as a private good when the legislators are impatient. However, as legislators become more patient, the total allocation to the private good dimension declines. That result is explained by the fact that as legislators become more patient, collective-leaning legislators are more likely to reject the current proposal and wait for the next session to increase spending on the collective good. Given that observation, the return to exerting effort falls, and hence equilibrium effort levels decline for the particularistic-leaning legislators. Therefore, the total amount of private good allocated to the distributive-minded legislators decreases. This leads to decreases in the equilibrium effort levels for collective-leaning legislators, who are in the majority.

Example 2 In the next example, we present the equilibrium effort levels and allocations of private and public goods depending on the legislators' patience level (δ) for two legislatures of different sizes. We assume that the majority of legislators are collective-leaning. Therefore, if a collective-leaning legislator is recognized, then the game ends with the equilibrium allocations $x^* = 0$ and $y^* = 1$. Tables 2 and 3 present cases in which a particularistic-leaning legislator $j \in B$ is recognized. Note that the marginal cost of effort is fixed at k=1 for all legislators.

Table 2: Equilibrium effort levels and allocation decisions with a small majority of collective-leaning legislators

	$m=4,\ n=7$	$m=4,\ n=7$	$m=4,\ n=7$	$m=4,\ n=7$
	$\delta=0.1$	$\delta=0.4$	$\delta=0.6$	$\delta=0.9$
e_i^*	0.151303	0.11951	0.091373	0,029763
e_j^*	0.051481	0.043581	0.035189	0,012688
y^*	0.06587	0.291317	0.474518	0,83786
x_j^*	0.93413	0.708683	0.525482	0,16214

Table 3: Equilibrium effort levels and allocation decisions with a large majority of collective-leaning legislators

	$m=14,\ n=15$	$m=14,\ n=15$	$m=14,\ n=15$	$m=14,\ n=15$
	$\delta=0.1$	$\delta=0.4$	$\delta=0.6$	$\delta=0.9$
e_i^*	0.057068	0.040374	0.028076	0,00751
e_j^*	0.057453	0.041510	0.029295	0.0080
y^*	0.088176	0.366515	0.565054	0.899571
x_j^*	0.911824	0.633485	0.434946	0.100429

Even though the majority consists of collective-leaning legislators in both Table 2 and Table 3, both group members exert different levels of effort depending on the legislature's size. The main discrepancy arises from the proposer power of the majority. The majority includes 57.14% of legislators in Table 2 while the percentage is 93.3% in Table 3. The robustness of the majority causes the minority to exert less effort, which is the main reason for different equilibrium allocations of public and private goods in Table 2 versus Table 3. It is obvious that the robustness of the majority in Table 2 is less than in Table 3. In both examples, as the discount rate increases, equilibrium effort levels do not only decline, but our model also converges to the legislative bargaining model with an exogenous recognition process. However, in Table 3, where the robustness of the majority is almost perfect, we observe very low investment in the recognition process by collective-leaning legislators since it is very likely that a collective-leaning legislator would be recognized as the proposer in the first session. Therefore, we can state that as the robustness of the majority and the patience level of legislators increase, wasteful resource spending to attain proposer power falls.

In the next proposition, we present the equilibrium level of collective and distributive allocation decisions when the majority of the legislature is composed of particularistic-leaning legislators.

Proposition 8 *Suppose the majority of the legislature is composed of particularistic-leaning legislators ($n-m \geqslant \frac{n+1}{2}$). If a collective-leaning $i \in A$ is recognized, then legislator i proposes*

$$x = \frac{2(n-m)\delta e_j^*(1-k(me_i^*+(n-m)e_j^*))}{2(n-m)(me_i^*+(n-m)e_j^*)-\delta(n+1-2m)me_i^*}$$

as a private good to $\dfrac{n+1-2m}{2}$ particularistic-leaning legislators selected at random and spends

$$y = 1-\left(\frac{n+1-2m}{2}\right)x = 1-\frac{\delta(n+1-2m)(n-m)e_j^*(1-k(m(e_i^*-e_j^*)+ne_j^*))}{\delta m(n+1-2m)e_i^*-2(n-m)(m(e_i^*-e_j^*)+ne_j^*)}$$

on the public good. If a particularistic-leaning $j \in B$ is recognized, then legislator j proposes

$$x = \frac{2(n-m)\delta e_j^*(1-k(me_i^*+(n-m)e_j^*))}{2(n-m)(me_i^*+(n-m)e_j^*)-\delta(n+1-2m)me_i^*}$$

to $\dfrac{n-1}{2}$ other particularistic-leaning legislators selected at random and keeps

$$x_j^j = 1-\left(\frac{n-1}{2}\right)x = 1-\frac{\delta(n-1)(n-m)e_j^*(k(m(e_i^*-e_j^*)+ne_j^*)-1)}{\delta m(n+1-2m)e_i^*-2(n-m)(m(e_i^*-e_j^*)+ne_j^*)}$$

for himself with no public good investment in the equilibrium.

Similar to Proposition 7, the proposal is implemented in the first session. Since we are dealing with an extreme case in which the majority consists of particularistic-leaning legislators, the collective-leaning ones must offer a mixed proposal containing both dimensions in order to form a winning coalition. On the other hand, any recognized particularistic-leaning legislator offers a distributive proposal that contains no public good spending. Proposition 8 implies that we can observe an equilibrium decision consisting of both ideological and distributive dimensions even though a majority is formed by particularistic-leaning legislators. Even in that extreme case, we can observe spending on collective goods in the equilibrium. Therefore, we also expect to see equilibrium decisions containing both public and private good dimensions as the political ideologies of legislators become closer, $|\alpha_p - \alpha_c|$ decreases. Note also that the amount of private good allocated to the unrecognized particularistic-leaning legislators who are in the winning coalition is the same regardless of the type of the recognized proposer in the first session because legislators' valuations of private good spending are at the extreme ideological positions.

6 Concluding remarks

In this paper, we combine the literature of legislative bargaining with the rent-seeking contest to analyze a sequential legislative bargaining game over private and public goods with endogenous recognition of a proposer. We show that legislators can exert effort to become the proposer and make proposals in both private good and public good dimensions depending on legislative preferences. When all legislators are collective-leaning, no conflict of interest exists in the legislature and so exerting effort to be recognized as the proposer becomes an unproductive activity. However, if legislators prefer to obtain private benefits, they exert effort in the equilibrium, and we thus observe wasteful resource spending in the recognition process. While the optimal effort levels are positively correlated with the valuation of private good spending, α, they are negatively correlated with the number of legislators and legislators' level of patience. In an asymmetric legislature, we show that any type of legislator exerts positive effort in equilibrium regardless of the legislature's composition. Moreover, we can observe equilibrium investment in collective goods even when a majority is formed by particularistic-leaning legislators; likewise, it may be possible to observe distributive policies when the majority of legislators are collective-leaning.

Our findings show how the rent-seeking behavior of legislators changes given that collective goods are also on the policy-making agenda. We explicitly model the trade-off between collective goods and particularistic goods as in Volden and Wiseman (2007). Even though we get similar results related to the provision of public goods, our findings on the characterization of legislators, cutoff points between the mixed and particularistic parts of the equilibrium, and the equilibrium level of public good provision in the mixed equilibrium are different owing to an endogenized costly recognition process. We observe less investment in the public good dimension than Volden and Wiseman (2007). In our model, the proposer spends less on the public good, hence keeping more resources for himself because if the proposal is rejected, the legislator must invest in the recognition process during the next legislative session. The other legislators are willing to approve the first session's proposal with less public good investment than in Volden and Wiseman (2007) to avoid the additional cost of the effort they would need to bear in the next session if the proposal is rejected. Therefore, incorporating an endogenized costly proposer recognition process into the model reduces total spending on the collective good in the mixed part of the equilibrium.

In our model, the legislators' efforts are unproductive, and thus socially undesirable, as in the model of Yildirim (2007), which is one of our model's benchmarks. A legislator basically exerts effort to capture proposer power and to affect the policy-making process so that he will be favored by the approved decision. Unlike that of Yildirim (2007), our model admits the possibility of collective spending that produces broader benefits than particularistic spending. The possibility of collective spending reduces the level of effort allocated to the proposer recognition process unless we are in an equilibrium in which legislators' valuations of particularistic spending exceed a certain threshold. Therefore, we observe less wasteful resource allocations to the recognition process than in Yildirim's (2007) model, implying that the existence of a collective-good spending dimension reduces the proposer's power.

We present a model of legislative bargaining over both private and public goods with endogenous proposer recognition. Several possible extensions of our work are left for future endeavors. Our model assumes that disagreement values are homogeneous. Heterogeneous and/or endogenous disagreement values can certainly affect the strategic interaction between legislators and the proposer's power can be explored. Another direction for further research is related to making the collective surplus endogenous as well. In our model, investment in the recognition process is unproductive. It would be interesting to explore the consequences of allowing legislators to choose to exert effort in productive activities that would increase the collective surplus and in unproductive activities in the recognition process, and then to study results related to proposer power and equilibrium legislative outcomes. Moreover, the predictions of our model can be tested experimentally in the lab. See, among others, McKelvey (1991), Miller and Vanberg (2013), Bradfield and Kagel (2015), and Miller et al. (2018) for experimental investigations of the Baron-Ferejohn model in different frameworks.

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Appendix

The main idea of for the proofs of Proposition 1 and 2 relies on the fact that there are mainly three types of decisions that can be supported in the symmetric equilibrium for different levels of α, as in Volden and Wiseman (2007). Before starting the proofs, we state a lemma that is relatively standard in the rent-seeking literature. The result is also stated and proved by Yildirim (2007). We provide a sketch of the proof for completeness.

Lemma 1: *In the one-shot legislative game in which* $p_i(e) = \begin{cases} \frac{e_i}{\sum_{j \in N} e_j} & \text{if } e \neq \vec{0}; \\ \frac{1}{n} & \text{if } e = \vec{0} \end{cases}$ *and the recognized legislator receives an exogenous prize* $\Pi_i > 0$*, there exists a unique pure strategy equilibrium such that* $e_i \geqslant e_j$ *whenever* $\frac{k_i}{\Pi_i} \leq \frac{k_j}{\Pi_j}$ *for some* $i, j \in N$*. Moreover, if* $\Pi_i = \Pi$ *and* $k_i = k$ *for all* $i \in N$*, then* $e_i = e_j > 0$ *and thus* $p_i(e) = p_j(e) = \frac{1}{n}$ *for all* $i, j \in N$*.*

Proof First, notice that $e_i = 0$ for all $i \in N$ cannot be an equilibrium. The expected equilibrium payoff for legislator i satisfies the following program: $v_i = \max_{e_i}\{p_i(e)\Pi_i - k_i e_i\}$. Taking the derivative of the terms inside the brackets yields $\frac{\partial p_i(e)}{\partial e_i}\Pi_i - k_i \leqslant 0$ ($= 0$ if $e_i > 0$).[6] This implies $\frac{\sum_{j \neq i} e_j}{(\sum_j e_j)^2} = \frac{1-p_i}{\sum_j e_j} = \frac{k_i}{\Pi_i}$, which is equivalent to $\sum_j e_j = \frac{(n-1)}{\sum \frac{k_i}{\Pi_i}}$. Therefore, $p_i = 1 - (n-1)\frac{\frac{k_i}{\Pi_i}}{\sum_j \frac{k_j}{\Pi_j}}$. Note that if $\Pi_i = \Pi$ and $k_i = k$ for all $i \in N$, then $p_i = \frac{1}{n}$. For a proof of uniqueness for the general case, see Szidarovsky and Okuguchi (1997). $\square$

Proof (Proposition 1) If legislator i is recognized in session t, he chooses a proposal according to the following program: $d^i \in argmax_d\ \alpha x_i^i + (1-\alpha)y^i$ subject to $\alpha x_j^i + (1-\alpha)y^i = \delta v_j$ for all $j \in C$ where $\mid C \mid = \dfrac{n+1}{2}$ and $y^i + \sum_{k \in N} x_k^i \leq 1$. Notice that $\alpha x_i^i + (1-\alpha)y^i = (2\alpha - 1)x_i^i + (1-\alpha) + (\alpha - 1)\sum_{k \in C \setminus \{i\}} x_k^i$. This implies that $d^i = (1, 0, \ldots, 0)$ if $\alpha \in [0, \dfrac{1}{2}]$. Therefore, being recognized does not affect the expected payoffs since the approved proposal involves only the public good dimension. Then, legislator i's continuation payoff is $v_i = max_{e_i} p_i(e)(1-\alpha) + (1-p_i(e))(1-\alpha) - ke_i$. This implies $e_i = 0\ \forall\ i \in N$. Hence, legislators

[6] It is easy to see that the second order condition holds.

do not exert effort because it is costly and recognition probabilities are identical. Therefore, the legislator who is recognized in the first session will put all resources towards the public good dimension and this decision will be approved by all legislators. $\square$

Proof (Proposition 2) Suppose legislator i is recognized in the first session, and he keeps x_i^i for himself and invests $y^i = 1 - x_i^i$ in the public good dimension. Suppose this decision is rejected by a majority in the legislature. Let j be a member of the coalition who voted no in the first session. He then expects that a decision can be approved in the second session if and only if $x_j^j = x_i^i - \epsilon_j$ and $y^j = y^i + \epsilon_j$, where $\epsilon_j \in (0, 1)$ in the mixed part of the equilibrium. By stationarity, legislator $j \neq i$ would make the same decision if he is recognized in the first session and the decision is accepted. This implies that legislator i's decision is not optimal and cannot be a part of the equilibrium since he is better off making the same proposal as legislator j in the first session. Therefore, the game ends in the first session and $x_i^i = x_j^j = x$ for all $i, j \in N$. The legislative game can then be thought as a one-shot game with a fixed prize ($\Pi = \alpha x$).[7] From Lemma 1, $p_i = \frac{1}{n}$ and $e_i = e_j = e = \dfrac{(n-1)}{n^2 k}\Pi > 0$ for all $i, j \in N$. In the voting stage, non-proposer j votes yes if and only if $(1-\alpha)(1-x) \geqslant \delta p_j(e)\alpha x + \delta(1-\alpha)(1-x) - \delta k e_j$. That is, $(1-\alpha)(1-x) \geqslant \delta p_j(e)\alpha x + \delta(1-\alpha)(1-x) - \delta\dfrac{(n-1)}{n^2}\alpha x$. This implies that $x \leq \dfrac{(1-\alpha)(1-\delta)n^2}{\delta\alpha + (1-\alpha)(1-\delta)n^2}$. Then, to maximize his utility, legislator i sets $x = \dfrac{(1-\alpha)(1-\delta)n^2}{\delta\alpha + (1-\alpha)(1-\delta)n^2}$ and $y = \dfrac{\delta\alpha}{\delta\alpha + (1-\alpha)(1-\delta)n^2}$. Given these equilibrium decisions for the mixed part of the equilibrium, the equilibrium level of effort exerted by a legislator can be written as $e = \dfrac{(n-1)}{n^2 k}\Pi = \dfrac{(n-1)}{n^2 k}\alpha x = \dfrac{(n-1)}{k}\left(\dfrac{\alpha(1-\alpha)(1-\delta)}{\delta\alpha + (1-\alpha)(1-\delta)n^2}\right) > 0$. Note that this decision is approved unanimously in the first session. Legislator i's utility is then given by $U_I = \Pi + (1-\alpha)y - ke = \Pi(2 - \dfrac{1}{\alpha} - \dfrac{n-1}{n^2}) + 1 - \alpha$.

In the particularistic dimension of the equilibrium, i.e. $\alpha \in (\alpha_p, 1]$, suppose legislator i keeps $x_i^i = x' = 1 - \theta(\frac{n-1}{2})$ for himself, invests $y^i = 0$ in the public good, and divides the remaining surplus $1 - x'$ between $\frac{n-1}{2}$ legislators at random. By stationarity, the legislative

[7] Note that each legislator gets $(1-\alpha)(1-x)$ even if he is not selected.

game can be thought as a one-shot game with a prize ($\Pi = \alpha x'$). From Lemma 1, $e_i = e_j = \dfrac{(n-1)}{n^2 k}\Pi' > 0$ and thus $p_i = p_j = \frac{1}{n}$ for all $i, j \in N$. By symmetry, $x_i^i = x_j^j = x' = 1 - \left(\frac{n-1}{2}\right)\theta$ for all $i, j \in N$. This implies that the model reduces to that of Baron and Ferejohn (1989). We state the rest of the proof for completeness. In the voting stage, non-proposer j votes yes if and only if $\alpha\theta \geqslant \delta p_j(e)\alpha(1 - \frac{n-1}{2}\theta) + \delta(1 - p_j(e))\frac{1}{2}\alpha\theta - \delta k e_j$. This implies $\theta \geqslant \dfrac{2\delta}{2n^2 - \delta(n-1)^2}$. Then, to maximize utility, legislator i sets $\theta = \dfrac{2\delta}{2n^2 - \delta(n-1)^2}$. Note that this decision is approved by a majority in the first session and the optimal effort level directly becomes (from Lemma 1) $e = \dfrac{\alpha(n-1)}{n^2 k}\left(1 - \dfrac{(n-1)\delta}{2n^2 - \delta(n-1)^2}\right) > 0$. Legislator i's utility is then given by $U_{II} = \Pi' - ke' = \Pi'(1 - \dfrac{n-1}{n^2})$. We now show how to find the cut-off value α_p by comparing the stage utility levels of the proposer. If $\alpha \in [\alpha_p, 1]$, then we should have $U_{II} \geqslant U_I$. Then α_p is given by solving the following equation: $U_{II} = \Pi' - ke' = \Pi'(1 - \frac{n-1}{n^2}) \geq U_I = \Pi(2 - \dfrac{1}{\alpha} - \dfrac{n-1}{n^2}) + 1 - \alpha$. This implies that the cutoff point is $\alpha_p = \dfrac{n[1 - 2n - \delta n^2 + (\delta - 1)n^3]}{\delta(n^4 - 2n^2 + 2n - 1) - n(n^3 + 2n^2 + 1)}$. □

Proof (Proposition 3) Note that $y = \dfrac{\delta\alpha}{\delta\alpha + (1-\alpha)(1-\delta)n^2}$ in the mixed part of the equilibrium. It is then easy to see that $\dfrac{\partial y}{\partial \delta} > 0$, $\dfrac{\partial y}{\partial n} < 0$, and $\dfrac{\partial y}{\partial \alpha} > 0$. □

Proof (Proposition 4) Note that $e = \dfrac{(n-1)}{k}\left(\dfrac{\alpha(1-\alpha)(1-\delta)}{\delta\alpha + (1-\alpha)(1-\delta)n^2}\right)$ in the mixed part of the equilibrium. It is easy to see that $\dfrac{\partial e}{\partial \delta} < 0$ and $\dfrac{\partial e}{\partial k} < 0$. Moreover,

$$\frac{\partial e}{\partial n} = \frac{\alpha(1-\alpha)(1-\delta)}{k(\delta\alpha + (1-\alpha)(1-\delta)n^2)^2}[\delta\alpha - (1-\alpha)(1-\delta)(n^2 - 2n)].$$

This implies that $\dfrac{\partial e}{\partial n} < 0$ if $n > \sqrt{\dfrac{\delta\alpha + (1-\delta)(1-\alpha)}{(1-\delta)(1-\alpha)}} + 1$. Finally,

$$\frac{\partial e}{\partial \alpha} = \frac{(n-1)(1-\delta)}{k\left\{\delta\alpha + (1-\alpha)(1-\delta)n^2\right\}^2}\left\{-\delta\alpha^2 + (1-\alpha)^2(1-\delta)n^2\right\}.$$

If $n > \sqrt{\left(\frac{\delta}{1-\delta}\right)\left(\frac{\alpha}{1-\alpha}\right)}$, then $\frac{\partial e}{\partial \alpha} > 0$. Therefore, the sufficient condition for both comparative statics results for the optimal effort level to hold is

$$n > \max\left\{\sqrt{\frac{\delta\alpha + (1-\delta)(1-\alpha)}{(1-\delta)(1-\alpha)}} + 1, \sqrt{\left(\frac{\delta}{1-\delta}\right)\left(\frac{\alpha}{1-\alpha}\right)}\right\}.$$

$\square$

Proof (Proposition 5) Note that $e' = \frac{\alpha(n-1)}{n^2 k}\left(1 - \frac{(n-1)\delta}{2n^2 - \delta(n-1)^2}\right)$ in the particularistic part of the equilibrium. It is then easy to see that $\frac{\partial e'}{\partial n} < 0$, $\frac{\partial e'}{\partial \delta} < 0$, $\frac{\partial e'}{\partial k} < 0$, and $\frac{\partial e'}{\partial \alpha} > 0$.

$\square$

Proof (Proposition 6) We show that legislators have incentives to exert positive effort when the other type of legislators exert zero effort.

Case 1 The majority consists of legislators who support public good provision, i.e. $m > \frac{n-1}{2}$. First, suppose that both types of legislators exert zero effort, i.e. $(e_i, e_j) = (0, 0)\ \forall\, i \in A$ and $\forall\, j \in B$. For any legislator $j \in B$, he takes x for himself and gives $1-x$ for the supporters of public good provision. In the critical voting stage, non-proposer legislator $i \in A$ will support the decision of any legislator $j \in B$ if and only if $(1-x) \geq \delta\left(\frac{m}{n} + \frac{(n-m)}{n}(1-x)\right)$. This implies $x = \frac{n - \delta n}{n - \delta n + \delta m}$ and $y = \frac{\delta m}{n - \delta n + \delta m}$. Then the expected utility for the legislators $j \in B$ is $EU_j = \frac{(n-m)}{n}\left(\alpha_p \frac{1}{n-m} x\right) + \frac{m}{n}\alpha_p 0 = \frac{1-\delta}{n - \delta n + \delta m}$. Similarly, the expected utility for the legislators $i \in A$ is $EU_i = \frac{m}{n} + \frac{\delta(n-m)m}{n(n - \delta n + \delta m)}$. Now we ask whether there exists an incentive for any legislator to exert positive effort when both types exert zero effort initially. If $e_i = 0\ \forall\, i \in A$, then any legislator $j \in B$ exerts $e_j > 0$ and a distributive legislator, $j \in B$, will be recognized with certainty. Then, to exert positive effort, we must have the following: $\frac{n - \delta n}{(n-m)(n - \delta n + \delta m)} - k e_j \geq \frac{1-\delta}{n - \delta n + \delta m}$. Then, $e_j \in \left(0, \frac{(1-\delta)m}{(n-m)(n - \delta n + \delta m)k}\right]$. It is clear that in this interval, we find that exerting positive effort makes legislator $j \in B$ better off while $e_i = 0 \; \forall \; i \in A$. If $e_j = 0 \; \forall \; j \in B$, any legislator $i \in A$ exerts $e_i > 0$ and any collective legislator, $i \in A$, will be recognized with certainty. Note that there exists only the ideological dimension in the equilibrium since the majority consists of legislators who support public good provision. Then, to exert positive effort, we must have the following: $1 - ke_i \geq \frac{m}{n} + \frac{\delta(n-m)m}{n(n-\delta n+\delta m)}$. Then, $e_i \in \left(0, \frac{(n-m)(1-\delta)}{(n-\delta n+\delta m)k}\right]$. Obviously, in this interval, we find that exerting positive effort makes legislator $i \in A$ better off while $e_j = 0 \; \forall \; j \in B$. Thus, exerting positive effort makes each type of legislator better off if the other group exerts zero effort. Therefore, we must have $(e_i, e_j) \neq (0,0) \; \forall \; i \in A$ and $\forall \; j \in B$. Then, for any $i \in A$,

$$EU_i = \frac{me_i}{me_i + (n-m)e_j} + \frac{(n-m)e_j}{me_i + (n-m)e_j}\left(\frac{\delta e_i[m(1-ke_i) - ke_j(n-m)]}{me_i + (1-\delta)(n-m)e_j}\right) - ke_i,$$

and for any $j \in B$,

$$EU_j = \frac{e_j}{me_i + (n-m)e_j}\left(\frac{(1-\delta+\delta ke_i)(me_i + (n-m)e_j)}{me_i + (1-\delta)(n-m)e_j}\right) - ke_j.^{8}$$

Given the expected payoffs, it is clear that $\exists \; (e_i, e_j) \neq (0,0)$ such that $EU_i(e_i, e_j = \epsilon) > EU_i(0, e_j = \epsilon)$ and $EU_j(e_i = \epsilon', e_j) > EU_j(e_i = \epsilon', 0)$, where ϵ and ϵ' are very small positive real numbers.

Case 2 The majority consists of legislators who support private good provision, i.e. $m < \frac{n-1}{2}$. First, suppose that both types of legislators exert zero effort, i.e. $(e_i, e_j) = (0,0) \; \forall \; i \in A$ and $\forall \; j \in B$. Any recognized legislator $j \in B$ gives x'' particularistic goods to $\frac{n-1}{2}$ other legislators who support private good allocation and keeps $x^P = 1 - \left(\frac{n-1}{2}\right)x''$. Any recognized legislator $i \in A$ gives x' particularistic goods to $\frac{n+1-2m}{2(n-m)}$ distributive legislators and puts $y = 1 - \left(\frac{n+1-2m}{2(n-m)}\right)x'$ towards the collective. In the critical voting stage, if a collective one is recognized, the non-proposer distributive legislator votes "Yes" if

[8] The derivations of EU_i and EU_j are explicitly stated in the proof of Proposition 7.

and only if

$$\alpha_p x^{'}+(1-\alpha_p)y \geq \delta\left\{\frac{1}{n}x^P+\left(\frac{n-m-1}{n}\right)\frac{(n-1)}{2(n-m-1)}\alpha_p x^{''}+\frac{m}{n}\left[\frac{(n+1-2m)}{2(n-m)}\alpha_p x^{'}+(1-\alpha_p)y\right]\right\}.$$

Furthermore, in the critical voting stage, if a distributive one is recognized, the non-proposer distributive legislator votes "Yes" if and only if

$$\alpha_p x^{''} \geq \delta\left\{\frac{1}{n}x^P+\left(\frac{n-m-1}{n}\right)\frac{(n-1)}{2(n-m-1)}\alpha_p x^{''}+\frac{m}{n}\left[\frac{(n+1-2m)}{2(n-m)}\alpha_p x^{'}+(1-\alpha_p)y\right]\right\}.$$

Also note that $\alpha_p = 1$ and in equilibrium we must have $\alpha_p x^{'} + (1-\alpha_p)y = \alpha_p x^{''}$, and since $\alpha_p = 1$, we have $x^{'} = x^{''}$. Optimality then requires that

$$x^{'} = \frac{2\delta(n-m)}{2(n-m)n-\delta m(n+1-2m)} = x^{''},$$

$$x^P = \frac{2(n-m)n-\delta m(n+1-2m)-\delta(n-m)(n-1)}{2(n-m)n-\delta m(n+1-2m)},$$

and $y = \dfrac{2(n-m)n-\delta(m+1)(n+1-2m)}{2(n-m)n-\delta m(n+1-2m)}$. Then the expected utility for legislators $i \in A$ is

$$EU_i^0 = \frac{m}{n}(1-\alpha_c)y+\frac{(n-m)}{n}\alpha_c 0 = \frac{2mn(n-m)-\delta m(m+1)(n+1-2m)}{2(n-m)n^2-\delta mn(n+1-2m)}.$$

Similarly, the expected utility for legislators $j \in B$ is

$$EU_j^0 = \frac{m(n+1-2m)}{2n(n-m)}\alpha_p x^{'}+\frac{1}{n}\alpha_p x^P+\frac{(n-m-1)}{n}\frac{(n-1)}{2(n-m-1)}\alpha_p x^{''} = \frac{2(n-m)}{2(n-m)n-\delta m(n+1-2m)}.$$

Now we check whether there exists any incentive for any legislator to exert positive effort when the other type exerts zero effort initially. If $e_i = 0 \; \forall \; i \in A$, any legislator $j \in B$ exerts $e_j > 0$ and a distributive legislator, $j \in B$, will be recognized directly. Then, to exert positive effort, the following equation must hold:

$$\frac{x^P}{(n-m)} + \frac{(n-m-1)}{(n-m)}\left(\frac{(n-1)}{2(n-m-1)}x^{'}\right) - ke_j \geq EU_j^0 = \frac{2(n-m)}{2(n-m)n - \delta m(n+1-2m)}.$$

Then, when $e_j \in \left(0, \frac{2(n-m)m - \delta m(n+1-2m)}{kB(n-m)}\right]$ where $B = 2(n-m)n - \delta m(n+1-2m)$, the distributive legislator exerts positive effort while the collective ones are inactive. If $e_j = 0 \; \forall \; j \in B$, then any legislator $i \in A$ exerts $e_i > 0$ and collective ones are recognized directly. Then, to exert positive effort, the following equation must hold:

$$\frac{2(n-m)n - \delta(m+1)(n+1-2m)}{2(n-m)n - \delta m(n+1-2m)} - ke_i \geq \frac{m}{n}(1-\alpha_c)y = EU_i^0 = \frac{2mn(n-m) - \delta m(m+1)(n+1-2m)}{2(n-m)n^2 - \delta mn(n+1-2m)}.$$

When $e_i \in \left(0, \frac{C}{kD}\right]$ where $C = 2(n-m)^2 n - \delta(m+1)^2(n+1-2m)$ and $D = 2(n-m)n^2 - \delta mn(n+1-2m)$, then the collective legislator will exert positive effort while the distributive ones are inactive. As a result, each type of legislator becomes better off by exerting positive effort if the other type of legislator exerts zero effort. Thus, we must have $(e_i, e_j) \neq (0,0) \; \forall \; i \in A$ and $\forall \; j \in B$. Then, for any $i \in A$,

$$EU_i^{'} = \left(\frac{me_i}{me_i + (n-m)e_j}\right)(1-\alpha_c)\left\{1 - \left(\frac{n+1-2m}{2}\right)\eta\right\} - ke_i,$$

and for any $j \in B$,

$$EU_j^{'} = \left(\frac{me_i}{me_i + (n-m)e_j}\right)\alpha_p \eta + \left(\frac{e_j}{me_i + (n-m)e_j}\right)\alpha_p \eta^P + \frac{(n-1)e_j}{2(me_i + (n-m)e_j)}\alpha_p \nu - ke_j.$$ [9]

Given the expected payoffs, it is clear to see that $\exists \; (e_i, e_j) \neq (0,0)$ such that $EU_i^{'}(e_i, e_j = \epsilon) > EU_i^{'}(0, e_j = \epsilon)$ and $EU_j^{'}(e_i = \epsilon^{'}, e_j) > EU_j^{'}(e_i = \epsilon^{'}, 0)$, where ϵ and $\epsilon^{'}$ are very small positive real numbers. □

[9] The derivations of $EU_i^{'}$ and $EU_j^{'}$ are explicitly stated in the proof of Proposition 8.

Proof (Proposition 7) A collective legislator is recognized with the probability $\dfrac{me_i}{me_i + (n-m)e_j}$. If a collective one is recognized, decision d is accepted by the majority and the game ends in the first session with stage utilities $U_i = 1 - ke_i$ and $U_j = -ke_j$. A distributive legislator is recognized with the probability $\dfrac{(n-m)e_j}{me_i + (n-m)e_j}$. In the critical voting stage, a non-proposer collective legislator will say "Yes" if and only if

$$(1-\alpha_c)(1-x) \geq \delta \left(\frac{me_i}{me_i + (n-m)e_j}(1-\alpha_c) + \frac{(n-m)e_j}{me_i + (n-m)e_j}(1-\alpha_c)(1-x) - ke_i \right).$$

Note that $\alpha_c = 0$. Therefore, we have $x = \frac{(1-\delta+\delta ke_i)(me_i+(n-m)e_j)}{me_i+(1-\delta)(n-m)e_j}$ and $y = \frac{\delta e_i[m(1-ke_i)-ke_j(n-m)]}{me_i+(1-\delta)(n-m)e_j}$. Moreover, the game ends in the first session since the distributive legislator gives the amount that satisfies the continuation value of collective legislators, which makes them indifferent between saying yes or no. Now we show that the optimal effort levels e_i^* and e_j^* exist. Indeed, the intersection point(s) of best response correspondences for each group of legislators is not an empty set. The expected utility for the distributive legislator, denoted by EU_i, is

$$EU_i = \frac{me_i}{me_i + (n-m)e_j} + \frac{(n-m)e_j}{me_i + (n-m)e_j} \left(\frac{\delta e_i[m(1-ke_i) - ke_j(n-m)]}{me_i + (1-\delta)(n-m)e_j} \right) - ke_i.$$

Similarly, the expected utility for the distributive, denoted by EU_j, is

$$EU_j = \frac{e_j}{me_i + (n-m)e_j} \left(\frac{(1-\delta+\delta ke_i)(me_i + (n-m)e_j)}{me_i + (1-\delta)(n-m)e_j} \right) - ke_j.$$

Note that by Proposition 6, we have $(e_i, e_j) \neq (0,0)$, and thus we do not have any possible continuity problems. Moreover, e_i and e_j are bounded above because of the direct cost-benefit analysis. Then, by Debreu (1952), Fan (1952), and Glicksberg (1952),[10] there exists (e_i^*, e_j^*) such that the intersection of two best response correspondences are not an empty set. □

Proof (Proposition 8) Collective and distributive legislators are recognized with the prob-

[10]See Fudenberg and Tirole (1991) for more on the existence theorem.

abilities $\dfrac{me_i}{me_i+(n-m)e_j}$ and $\dfrac{(n-m)e_j}{me_i+(n-m)e_j}$, respectively. If a collective one is recognized, he gives η private good to $\left(\dfrac{n+1-2m}{2}\right)$ distributive legislators selected at random and puts $y=1-\left(\dfrac{n+1-2m}{2}\right)\eta$ towards the public good. If a distributive legislator is recognized, then he gives ν private good to $\left(\dfrac{n-1}{2}\right)$ other distributive legislators and keeps $\eta^P=1-\left(\dfrac{n-1}{2}\right)\nu$ for himself. Therefore, the expected utilities for each type of legislator become

$$EU_i^{'}=\frac{me_i}{me_i+(n-m)e_j}(1-\alpha_c)\left[1-\left(\frac{n+1-2m}{2}\right)\eta\right]+\frac{(n-m)e_j}{me_i+(n-m)e_j}(1-\alpha_c)0-ke_i$$

and

$$EU_j^{'}=\frac{me_i}{me_i+(n-m)e_j}\alpha_p\eta+\frac{e_j}{me_i+(n-m)e_j}\alpha_p\eta^P+\left(\frac{(n-m-1)e_j}{me_i+(n-m)e_j}\right)\left(\frac{n-1}{2(n-m-1)}\right)\alpha_p\nu-ke_j.$$

In the critical voting stage, a non-proposer distributive legislator votes yes if and only if

Case 1 If the collective legislator is recognized,

$$\alpha_p\eta+(1-\alpha_p)y\geq\delta\left[\frac{e_j}{me_i+(n-m)e_j}\alpha_p\eta^P+\left(\frac{(n-m-1)e_j}{me_i+(n-m)e_j}\right)\frac{(n-1)}{2(n-m-1)}\alpha_p\nu\right]$$
$$+\delta\left[\left(\frac{me_i}{me_i+(n-m)e_j}\right)\frac{(n+1-2m)}{2(n-m)}\alpha_p\eta-ke_j\right].$$

Case 2 If the distributive legislator is recognized,

$$\alpha_p\nu\geq\delta\left[\left(\frac{e_j}{me_i+(n-m)e_j}\right)\alpha_p\eta^P+\left(\frac{(n-m-1)e_j}{me_i+(n-m)e_j}\right)\frac{(n-1)}{2(n-m-1)}\alpha_p\nu\right]$$
$$+\delta\left[\left(\frac{me_i}{me_i+(n-m)e_j}\right)\left(\frac{n+1-2m}{2(n-m)}\right)\alpha_p\eta-ke_j\right].$$

Then, in the equilibrium, we must have $\alpha_p \eta + (1-\alpha_p)y = \alpha_p \nu$. Since $\alpha_p = 1$, we have $\eta = \nu = \dfrac{2(n-m)\delta e_j(1-k(me_i+(n-m)e_j))}{2(n-m)(me_i+(n-m)e_j)-\delta(n+1-2m)me_i}$ with $\eta^P = 1-\left(\dfrac{n-1}{2}\right)\eta$ and $y = 1-\left(\dfrac{n+1-2m}{2}\right)\eta$. Note that the game ends in the first session since both collective and distributive legislators offer the continuation value of other distributive legislators. This makes them indifferent to saying yes or no. By Proposition 6, we have $(e_i, e_j) \neq (0,0)$. Therefore, we do not have a possible continuity problem. Moreover, e_i and e_j are bounded above because of the direct cost-benefit analysis. Then, by Debreu (1952), Fan (1952), and Glicksberg (1952), there is a pair (e_i^*, e_j^*) such that the intersection of best response correspondences are not an empty set. $\square$