Q.1 Position vector of a particle as a function of time t is given by

$$\mathbf{r}(t) = \frac{4t}{\pi}\hat{\imath} + (5 + \cos 2t)\hat{\jmath} - \sqrt{2}\sin t\hat{k}$$

a) Find velocity $\mathbf{v}(t)$ and acceleration $\mathbf{a}(t)$ vectors of the particle.
b) Find magnitudes of $\mathbf{v}(t)$ and $\mathbf{a}(t)$ at the instant when the particle passes through the point $(1, 5, -1)$.
c) Find the equation of the line tangent to the trajectory of the particle at the point $(1, 5, -1)$.
d) Find an equation of the plane normal to the trajectory of the particle at the point $(1, 5, -1)$.

Q.2 Find the tangent, normal and binormal vector and compute the curvature and the torsion of the curve specified by

$$x(t) = a(1 + \cos t)\,, \quad y(t) = a\sin t\,, \quad z(t) = 2a\sin\frac{t}{2}\,.$$

This is called Viviani's curve.

Q.3
a) Find the directional derivative of the scalar field $\varphi(x, y, z) = x^2 + \sin y - xz$, in the direction of the vector $\mathbf{A} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ at the point $\left(1, \frac{\pi}{2}, -3\right)$.

b) In which direction does the scalar field $\varphi(x, y, z) = z\sin y - xz$ increases most rapidly at the point $\left(2, \frac{\pi}{2}, -1\right)$.

Q.4 Compute the diverge and the curl of the following vector fields:

a) $\mathbf{r}(t) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, b) $\mathbf{V}(t) = x^2y\hat{\imath} + y^2x\hat{\jmath} + xyz\hat{k}$, c) $\mathbf{V}(t) = x\sin y\hat{\imath} + \cos y\hat{\jmath} + xy\hat{k}$.

Q.5 Calculate the Laplacian $\nabla^2 = \nabla \cdot \nabla$ of the scalar fields

a) $\ln(x^2 + y^2)$,

b) $(x + y)^{-1}$.

Q.6 It is given that $\mathbf{r}(t) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. Compute

a) $\boldsymbol{\nabla} \times (\hat{k} \times \mathbf{r})$,

b) $\nabla \cdot \left(\frac{\mathbf{r}}{|\mathbf{r}|}\right)$,

c) $\nabla \cdot \left(\frac{\mathbf{r}}{|\mathbf{r}|}\right)$,

Q.7 Simplify the following expressions using index notation

a) $\nabla \times (\mathbf{U} \times \mathbf{V})$,

b) $\nabla(\mathbf{U} \cdot \mathbf{V})$,

c) $\nabla \cdot (\nabla\phi \times \nabla\psi)$,