Q.1 Expand the following functions in complete Fourier series of period 2π:

a.

$$f(x) = \begin{cases} 1 & 0 < x < \pi \\ 0 & -\pi < x < 0 \end{cases}$$

b.

$$f(x) = \begin{cases} -\frac{1}{2}(\pi + x) & -\pi \le x < 0 \\ \frac{1}{2}(\pi - x) & 0 < x \le \pi \end{cases}$$

c.

$$f(x) = |\cos x|\,, \quad 0 < x < \pi$$

d.

$$f(x) = \sinh x\,, \quad -\pi < x < \pi$$

e.

$$f(x) = \cosh x\,, \quad -\pi < x < \pi$$

Plot the periodic completion of these functions of period 2π on the entire real interval.

Q.2 Obtain the Fourier cosine series for the function $f(x) = \cos \alpha x$, where α is real number and and not an integer. Using the validity of the cosine series at the end point $x = \pi$ show that

$$\cot \pi\alpha = \frac{1}{\pi}\left(\frac{1}{\alpha} - \sum_{n=1}^{\infty} \frac{2\alpha}{n^2 - \alpha^2}\right).$$

Q.3 Find the Fourier transform of the functions

a.

$$f(x) = \begin{cases} |x| & |x| < 1 \\ 0 & |x| > 1 \end{cases}$$

b.

$$f(x) = \begin{cases} (1 - x^2) & |x| < 1 \\ 0 & |x| > 1 \end{cases}$$

b.

$$f(x) = \begin{cases} -2 & -3 < x < -1 \\ 2 & 1 < x < 3 \\ 0 & \text{elsewhere} \end{cases}.$$

d.

$$f(x) = e^{-\alpha x^2}\,, \quad -\infty < x < \infty\,.$$

e.

$$f(x) = e^{-|x|}\,, \quad -\infty < x < \infty\,.$$

In this case use the inverse transform to show that

$$\int_0^{\infty} \frac{\cos \alpha x}{\alpha^2 + 1} d\alpha = \frac{\pi}{2} e^{-|x|}\,.$$

f.

$$f(x) = \begin{cases} \sin x & 0 < x < \pi \\ 0 & \text{otherwise} \end{cases}$$

Now show using the Fourier integral and the inverse transform that the result in part f. can be written as

$$f(x) = \frac{1}{\pi} \int_0^\infty \frac{\cos \alpha x + \cos \alpha (x - \pi)}{1 - \alpha^2} d\alpha \,,$$

Q.4 Find the Fourier sine transform of the functions

a.

$$f(x) = \begin{cases} x & |x| < 1 \\ 0 & |x| > 1 \end{cases}$$

b.

$$f(x) = \begin{cases} \sin x & |x| < \frac{\pi}{2} \\ 0 & |x| > \frac{\pi}{2} \end{cases}$$

Q.5 Find the Fourier cosine transform of the functions

a.

$$f(x) = \begin{cases} |x| & |x| < 1 \\ 0 & |x| > 1 \end{cases}$$

b.

$$f(x) = \begin{cases} \cos x & |x| < \frac{\pi}{2} \\ 0 & |x| > \frac{\pi}{2} \end{cases}$$