PHYS-505: ELECTROMAGNETIC THEORY I

HOMEWORK II

Due 15.04.2014

Q1: A charge density $s(\theta)$ (θ is the polar angle) is placed on the surface of a spherical thin shell of radius R. Center of the shell is at the origin.
a) Find the electric field and potential inside and outside of the shell if $s(\theta)$ is constant and equal to s_0.
b) Find the electric field and potential inside and outside of the shell if $s(\theta) = s_0 cos 2\theta$

Q2: The potential on the surface of a sphere having radius R is given as $\Phi = V_0 cos^2\theta$, where V_0 is constant and θ is the polar angle. A point charge Q is placed at the center of the sphere (to the origin).
a) Find the electric potential everywhere using the solution of Laplace equation in spherical coordinates.
b) Find the charge density on the surface of the sphere.

Q3: A cylindrical pipe (Figure 1) of radius a is sawn lengthwise into two equal halves. A battery connected between the two halves establishes a potential difference of V_0 between the two halves. Use separation of variables to find the potential inside and outside the pipe.

Q4: Consider a potential problem in the half-space defined by $z \geq 0$, with Dirichlet boundary conditions on the plane $z = 0$ (and at infinity).
a) Write down the appropriate Green function $G(x, x^{'})$.
b) If the potential on the plane $z = 0$ is specified to be $\Phi = V$ inside a circle of radius a centered at the origin, and $\Phi = 0$ outside that circle, find an integral expression for the potential at the point P specified in terms of cylindirical coordinates (ρ, ϕ, z).
c) Show that, along the axis of the circle ($\rho = 0$), the potential is given by

$$\Phi = V(1 - \frac{z}{\sqrt{a^2 + z^2}}).$$

d) Show that at large distances ($\rho^2 + z^2 \gg a^2$) the potential can be expanded in a power series in $(\rho^2 + z^2)^{-1}$, and that the leading terms are

$$\Phi = \frac{Va^2}{2} \frac{z}{(\rho^2 + z^2)^{\frac{3}{2}}} [1 - \frac{3a^2}{4(\rho^2 + z^2)} + \frac{5(3\rho^2 a^2 + a^4))}{8(\rho^2 + z^2)^2} + \cdots]$$

Verify that the result of parts c and d are consistent with each other in their common range of validity.

Q5: Three point charges $(q, -2q, q)$ are located in a straight line with separation a and with the middle charge $(-2q)$ at the origin of a grounded conducting spherical shell of radius b, as indicated in Figure 2.

a) Write down the potential of the three charges in the absence of the grounded sphere. Find the limiting form of the potential as $a \to 0$, but the product $qa^2 = Q$ remains finite. Write this latter answer in spherical coordinates.

b) The presence of the grounded sphere of radius b alters the potential for $r < b$. The added potential can be viewed as caused by the surface-charge density induced on the inner surface at $r = b$ or by image charges located at $r > b$. Use linear superposition to satisfy the boundary conditions and find the potential everywhere inside the sphere for $r < a$ and $r > a$. Show that in the limit $a \to 0$,

$$\Phi(r, \theta, \phi) \to \frac{Q}{2\pi\epsilon_0 r^3}(1 - \frac{r^5}{b^5})P_2(cos\theta)$$

Q6: A line charge of length $2d$ with a total charge Q has a linear charge density varying as $(d^2 - z^2)$, where z is the distance from the midpoint. A grounded, conducting, spherical shell of inner radius $b > d$ is centered at the midpoint of the line charge.

a) Find the potential everywhere inside the spherical shell as an expansion in Legendre polynomials.

b) Calculate the surface-charge density induced on the shell.

c) Discuss your answers to parts a and b in the limit that $d \ll b$.

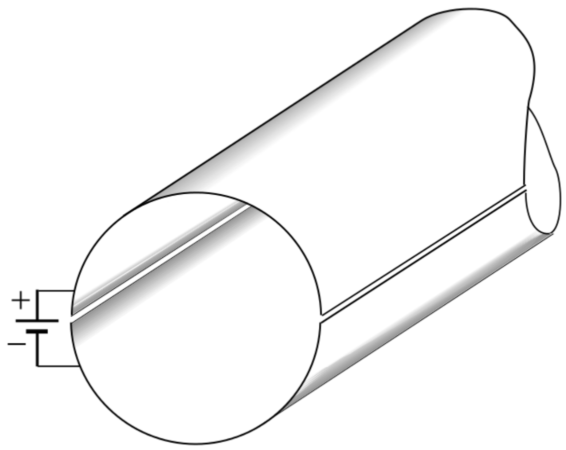

Figure 1:

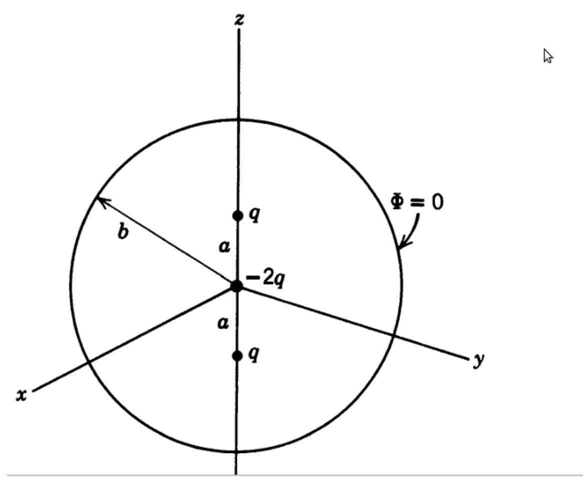

Figure 2: