METU Department of Mathematics
Math 349 Homework 5
Due 19 April 2018

1. Let X be a metric space, $f : X \to \mathbb{R}$ be a continuous function, and let A be a subset of X. Suppose that $f(a) = 0$ for all $a \in A$. Show that for $x \in X$ if $x \in \overline{A}$, then $f(x) = 0$.

2. Let A be a non-empty compact subset of a metric space X. For a given $x \in X$, prove that there is an element $a \in A$ such that $d(x, a) = d(x, A)$.

3. Find the interior, the boundary and the exterior of the following subsets of $\mathbb{R}^2$:
(a) $\{(x, y) : 1 \leq x^2 + y^2 < 9\}$.
(b) $\{(x, y) : 0 < x < y\}$.
(c) $\{(x, y) : x \text{ and } y \text{ irrational}\}$.
(d) $\{(x, y) : 0 < x = y + 1\}$.

4. Let (X, d) be a metric space. Define the distance of two nonempty subsets A and B of X by

$$d(A, B) = \inf\{d(a, b) : a \in A,\ b \in B\}.$$

(a) Give and example two closed sets A and B of some metric space with $A \cap B = \emptyset$ and $d(A, B) = 0$.
(b) If A and B are two nonempty compact subsets with $A \cap B = \emptyset$, then show that $d(A, B) > 0$.