

Math 349 Homework 4

Due 5 April 2018

1. Let $a_n = \frac{(-1)^n(n+1)}{n}$.

(a) Show that the sequence (a_n) is bounded. Find the limit inferior $\underline{\lim} a_n$ and the limit superior $\overline{\lim} a_n$.

(b) Find a subsequence (a_{k_n}) converging to $\underline{\lim} a_n$ and a subsequence (a_{l_n}) converging to $\overline{\lim} a_n$.

2. Let (a_n) be a sequence. Determine whether the given statements are true or false. Prove your answer.

(a) If (a_n) is convergent, then (a_n^2) is convergent.

(b) If (a_n^2) is convergent, then (a_n) is convergent.

3. Let (X, d) be a metric space, $x, y \in X$ with $x \neq y$.

(a) Show that the one-point set $\{x\}$ is closed.

(b) Show that there are open subsets U, V of X such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

(Do not use any theorem proved in class).

4. Prove that any discrete metric space (X, d) is complete.

5. Let X be a set and let d and d' be two **equivalent** metrics on X . Show that a subset A of X is open in (X, d) if and only if it is open in (X, d') .