Name:
Signature:

**METU, Spring 2018, Math 523.**

# Final

**1. (25pts)** Let $\alpha = \sqrt[3]{20}$ and $K = \mathbf{Q}(\alpha)$. You are given that $\mathcal{O}_K = \mathbf{Z}[1, \alpha, \alpha^2/2]$. Find the ideal prime decomposition of all ideals $p\mathbf{Z}$ that ramifies in the extension $K/\mathbf{Q}$.

**2. (25pts)** Consider the number fields $K = \mathbf{Q}(\sqrt{-6}\,)$ and $L = \mathbf{Q}(\sqrt{-6}, \sqrt{2}\,)$. Show that the extension $L/K$ is unramified, i.e. show that there is no prime ideal $\mathfrak{p} \subset \mathcal{O}_K$ that ramifies in $L$.

**3. (25pts)** Let $K = \mathbf{Q}(\sqrt{-21})$. Show that $\mathrm{Cl}(K) \cong \mathbf{Z}_2 \times \mathbf{Z}_2$.

**4. (25pts)** You are given that $u = 3 + \sqrt{10}$ is the fundamental unit of $\mathbf{Q}(\sqrt{10})$. For each one of the following Diophantine equations, answer the questions: 1) Does there exist any solution? 2) Is the number of solutions finite or infinite? 3) Is it possible to give a procedure to obtain each solution?

- $x^2 - 10y^2 = 1$

- $x^2 - 10y^2 = 3^{1001}$

- $x^2 - 10y^2 = 7^{1001}$

- $x^2 - 10y^2 = 31^{1001}$