METU, Spring 2012, Math 515.

Homework 5

(due May 9)

1. Let $A \subset B$ be rings, B integral over A. Let $\mathfrak{n} \subset B$ be a maximal ideal and set $\mathfrak{m} = \mathfrak{n} \cap A$. Give two examples such that
   - $B_{\mathfrak{n}}$ is integral over $A_{\mathfrak{m}}$,
   - $B_{\mathfrak{n}}$ is not integral over $A_{\mathfrak{m}}$.
2. Let $A \subset B$ be rings, B integral over A.
   - If $x \in A \cap B^{\times}$, then show that $x \in A^{\times}$.
   - Show that $\mathfrak{R}(A) = \mathfrak{R}(B) \cap A$ for Jacobson radicals of A and B.
3. Show that the ring of continous real-valued functions on $[0, 1]$ is not Noetherian.
4. Let M be an A-module and let N_1, N_2 be submodules of M. If M/N_1 and M/N_2 are Noetherian then prove that $M/(N_1 \cap N_2)$ is Noetherian.
5. Prove that $A/\mathrm{Ann}(M)$ is Noetherian if M is a Noetherian A-module.