METU, Spring 2012, Math 515.

Homework 4

(due April 23)

1. (4 points) Let A be a ring and let $S = \{f^n : n \geq 0\}$ for some $f \in A$. Show that the localisation $S^{-1}A$, sometimes denoted by A_f, is isomorphic to $A[X]/(Xf - 1)$.

2. (4 points) Show that an A-module homomorphism $\varphi : M \to N$ is surjective if and only if $\varphi_{\mathfrak{m}} : M_{\mathfrak{m}} \to N_{\mathfrak{m}}$ is surjective for each maximal ideal $\mathfrak{m} \subset A$.

3. (8 points) Let A be a ring and let $A[x]$ be the ring of polynomials in an indeterminate x with coefficients in A. For each ideal $\mathfrak{a} \subset A$, let $\mathfrak{a}[x]$ denote the set of polynomials with coefficients in $\mathfrak{a}$.

   (a) Prove that $f = \sum_{i=0}^{n} a_n x^n$ is nilpotent in $A[x] \Leftrightarrow a_0, a_1, \ldots, a_n$ are nilpotent in A.

   (b) If $\mathfrak{q}$ is a $\mathfrak{p}$-primary ideal in A, then show that $\mathfrak{q}[x]$ is a $\mathfrak{p}[x]$-primary ideal in $A[x]$.

4. (4 points) If $A = \mathbf{F}[x, y, z]$ then find a minimal primary decomposition for the ideal $\mathfrak{a} = (x^2, xy, xz, yz)$. Determine for each component if it isolated or embedded.