M E T U
Department of Mathematics

Group	Algebra I Midterm II	List No.

Code : *Math 503* Acad. Year : *2013* Semester : *Fall* Instructor : *Küçüksakallı* Date : *17/12/2013* Time : *13:40* Duration : *110 minutes*	Name : Last Name : Signature : 7 QUESTIONS ON 4 PAGES 30 TOTAL POINTS

1	2	3	4	5	6	7		

1. (6pts) Let S_n be the symmetric group on $I_n = \{1, 2, \ldots, n\}$.

- Show that S_n is generated by the $n-1$ transpositions $(12), (13), \ldots, (1n)$.

- If $\sigma = (i_1 i_2 \ldots i_r) \in S_n$ and $\tau \in S_n$, then $\tau\sigma\tau^{-1}$ is the r-cycle $(\tau(i_1)\tau(i_2)\ldots\tau(i_r))$.

- Show that S_n is generated by (12) and $(12\ldots n)$.

2. (4pts) Let G be an abelian group in which no element (except 0) has finite order. Prove or disprove the following claim: "Then G is a free abelian group."

3. (4pts) If a group G contains an element a having exactly two conjugates, then G has a proper normal subgroup $N \neq \langle e \rangle$.

4. (4pts) Let G be a group order 56. Show that G is solvable.

5. (4pts) Classify all groups of order 8 up to isomorphism.

6. (4pts) Let P be a Sylow p-subgroup of a nilpotent group G. Show that $P \triangleleft G$.

7. (4pts) Let x be a nilpotent element of the commutative ring R. (An element x in R is called nilpotent if $x^m = 0$ for some $m \in \mathbb{Z}^+$.)

- Prove that x is either zero or a zero divisor.

- Prove that $x + 1_R$ is a unit in R.