M E T U
Department of Mathematics

Group	Algebra I Midterm I	List No.

Code : *Math 503* Acad. Year : *2013* Semester : *Fall* Instructor : *Küçüksakallı* Date : *05/11/2013* Time : *13:40* Duration : *110 minutes*	Name : Last Name : Signature : 7 QUESTIONS ON 4 PAGES 30 TOTAL POINTS

1	2	3	4	5	6	7		

1. (4pts) Suppose that R and S are equivalence relations on A. Find a sufficient and necessary condition so that the formula $f([x]_R) = [x]_S$ defines a function from A/R to A/S?

2. (3pts) Show that the cardinality of a set A is strictly less than the cardinality of its power set $\mathcal{P}(\mathcal{A})$.

3. (4pts) If G is a finite abelian group of even order than show that there exists an element $g \in G$, not equal to the identity, such that $g^2 = e$.

4. (3pts) If p is a prime number, then show that the nonzero elements of $\mathbb{Z}_p$ form a group, denoted by $\mathbb{Z}_p^\times$, under multiplication. Determine if $\mathbb{Z}_{17}^\times$ is cyclic or not.

5. (8pts) Define $gHg^{-1} = \{ghg^{-1} | h \in H\}$. Define the normalizer of H in G to be the set $N_G(H) = \{g \in G | gHg^{-1} = H\}$.

- Show that $N_G(H)$ is a subgroup of G containing H.

- Prove that $N_G(H) = G$ if and only if $H \triangleleft G$.

- If H and K are subgroups of G and $H < N_G(H)$, then show that HK is a subgroup of G.

- If H and K are subgroups of G and $H < N_G(H)$, then show that HK/K is isomorphic to $H/(H \cap K)$.

6. (4pts) Let H and K be finite subgroups of G. Then prove that

$$|HK| = \frac{|H||K|}{|H \cap K|}.$$

7. (4pts) If $f : G \to H$ is a homomorphism, H is abelian and N is a subgroup of G containing $\mathrm{Ker}(f)$, then show that N is normal in G.