M E T U
Department of Mathematics

Group	Algebra I Final	List No.

Code : *Math 503* Acad. Year : *2013* Semester : *Fall* Instructor : *Küçüksakallı* Date : *20/01/2014* Time : *13:30* Duration : *150 minutes*	Name : Last Name : Signature : 7 QUESTIONS ON 4 PAGES 40 TOTAL POINTS

1	2	3	4	5	6	7		

1. (6pts) Let p be a prime and let $G = \mathbf{Z}_p \times \mathbf{Z}_{p^2} \times \mathbf{Z}_{p^3}$.

- Determine the number of cyclic subgroups of G of order p^3.

- Determine the number of noncyclic subgroups of G of order p^4.

2. (6pts) Let G be a finite group and H be a subgroup of G of order n. If H is the only subgroup of order n in G then show that H is normal in G.

3. (6pts) Let G be a finite group and let $H < G$ be a proper subgroup. Prove there exists an element of G that is not conjugate to an element of H. (Hint: First prove that there are at most $[G : H]$ subgroups of G that are conjugate to H.)

4. (5pts) Consider the ring $R = \mathbf{Z}[\sqrt{-3}]$. Find an element α in R which is irreducible but not prime. Show that R is not a unique factorization domain. (You can use the fact that $N(a + b\sqrt{-3}) = a^2 + 3b^2$ is multiplicative.)

5. (6pts) Let P be a prime ideal of a commutative ring with 1_R. Show that the prime ideals of R/P are in bijective correspondence with the prime ideals of R containing P.

6. (5pts) Consider the map $\phi : f(x, y) \mapsto f(x, x^2)$ from the polynomial ring $\mathbf{C}[x, y]$ to the polynomial ring $\mathbf{C}[x]$. Show that ϕ is a homomorphism of rings and determine its kernel.

7. (6pts) Let S be a multiplicative subset of an integral domain R such that $0_R \notin S$. If R is a principal ideal domain then show that the localization $S^{-1}R$ is a principal ideal domain.