Name and Surname:
Student Number:

Math 466 - Fall 2019 - METU

Quiz 6

The orthogonal group O_2 consists of two types of elements: rotations and reflections. The rotations form a normal subgroup, called SO_2. The elements in $O_2 \setminus SO_2$ are reflections and have order 2.

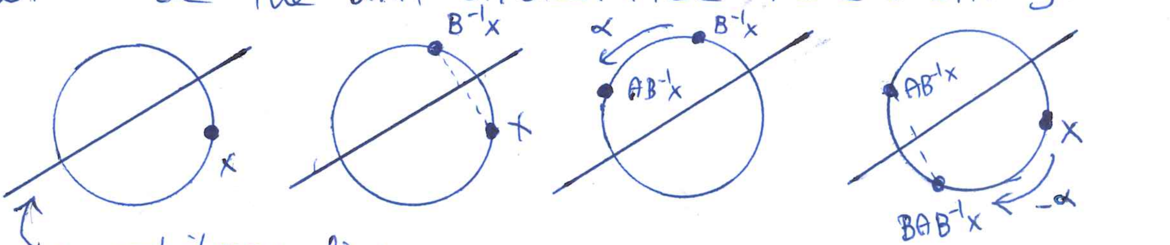
Write the general form of elements in SO_2 and $O_2 \setminus SO_2$.

If $A \in SO_2$, then $A = \begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}$ for some $a \in \mathbb{R}$

If $B \in O_2 \setminus SO_2$, then $B = \begin{bmatrix} \cos b & \sin b \\ \sin b & -\cos b \end{bmatrix}$ for some $b \in \mathbb{R}$

Fix $B \in O_2 \setminus SO_2$. For each $A \in SO_2$, define $\sigma_B(A) = BAB^{-1}$. Show that $\sigma_B(A) = A^{-1}$.

Let C be the unit circle. Pick $x \in C$ arbitrary.



an arbitrary line passing through the origin representing the mirror of B

Note that $BAB^{-1}x = A^{-1}x$ for all $x \in C$. We must have $BAB^{-1} = A^{-1}$

Show that $\sigma_B \in \mathrm{Aut}(SO_2)$.

The group SO_2 is abelian. The map $A \to A^{-1}$ is a group automorphism