Name and Surname:
Student Number:

Math 466 - Fall 2019 - METU

Quiz 5

For each of the following determine if there exists an isometry satisying the given conditions or not. Either express such an isometry in the normal form or explain why it does not exist.

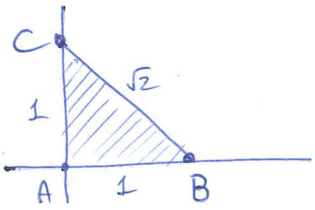
(1) $\boxed{\begin{array}{l} f(0,0)=(0,-1), \\ f(1,0)=(1,0), \\ f(0,1)=(-1,0). \end{array}}$

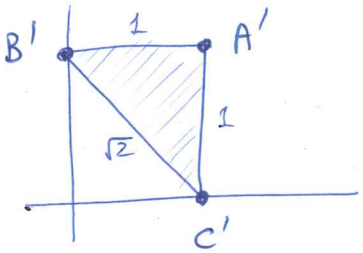
An isometry preserves the distance by definition.

If $A=(0,0)$ and $B=(1,0)$, then $\|A-B\|=1$. On the other hand $\|f(A)-f(B)\|=\sqrt{2}$. Thus f is not an isometry.

(2) $\boxed{\begin{array}{l} f(0,0)=(1,1), \\ f(1,0)=(0,1), \\ f(0,1)=(1,0). \end{array}}$

Set $A=(0,0)$, $B(1,0)$ and $C=(0,1)$, and $A'=f(A)$, $B'=f(B)$ and $C'=f(C)$.





We want to write $f=r^{\varepsilon}\cdot s\cdot t$. Orientation is preserved so $\varepsilon=0$. The rotation s must be of magnitude π, so $s^{-1}(x,y)=s(x,y)=(-x,-y)$. Now we must have $t(x,y)=(x-1,y-1)$. In summary $f=r^{0}\cdot s\cdot t$. We may also find

$$f(x,y)=s(t(x,y))$$
$$=s(x-1,y-1)=(1-x,1-y)$$