Name and Surname:
Student Number:

Math 466 - Fall 2019 - METU

Quiz 4

(a) Show that $M = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$ is an element of SO_3.

We have $M \cdot M^t = I$ and $\det(M) = 1$. Thus $M \in SO_3$. Recall that M gives a rotation of $\mathbb{R}^3$ fixing an axis passing thru the origin.

(b) Let $A = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$. Show that $M \cdot A = B$ and $M \cdot B = A$.

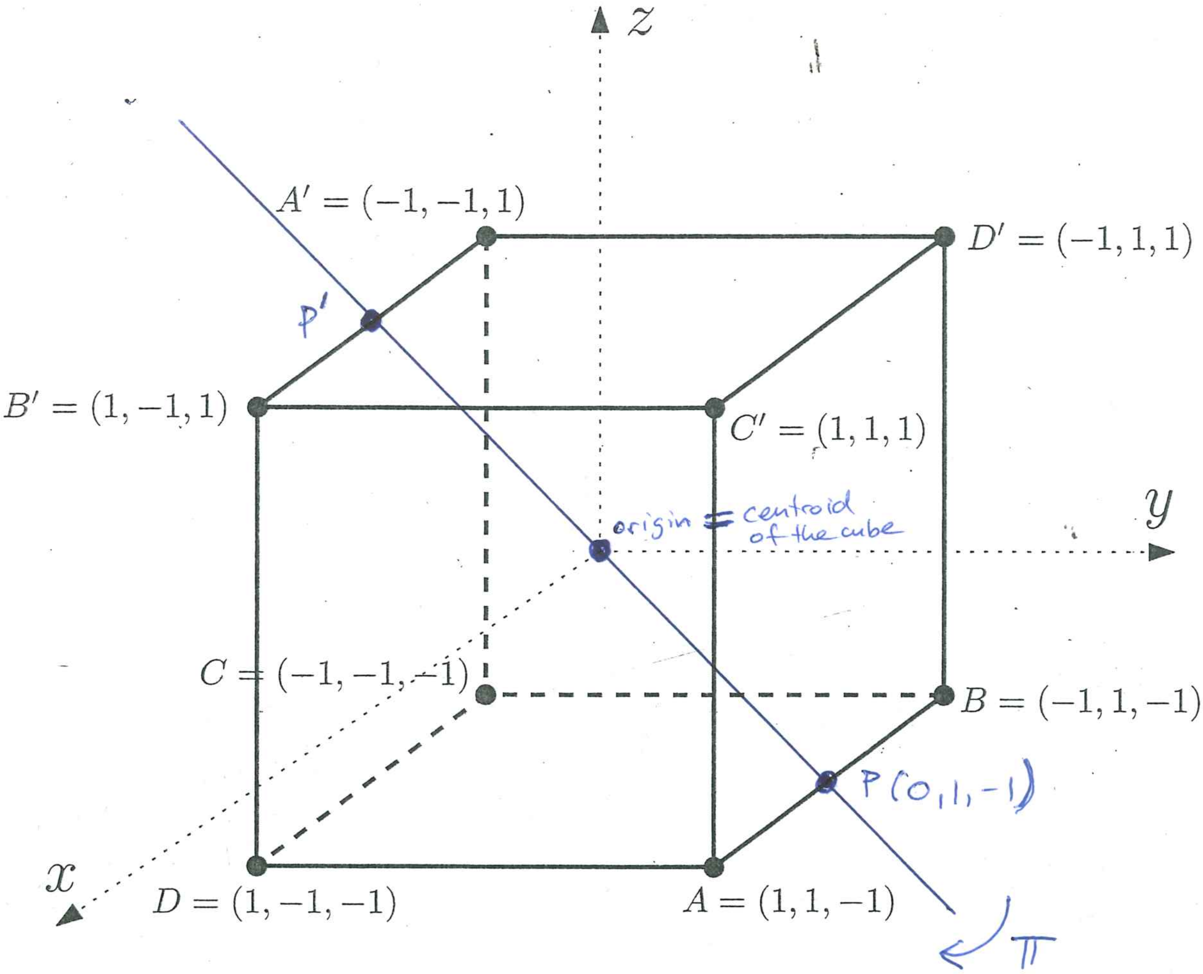
$$M \cdot A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} = B, \quad MB = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = A.$$

(c) Consider the cube in three dimensional space whose vertices are given by $(\pm 1, \pm 1, \pm 1)$. Describe the rotational action of M on this cube. (You may use the picture at the back.)

Consider the midpoint $P = (0, 1, -1)$ of A and B. We have $MP = P$. The axis of the rotation, produced by M, is the line with direction $\vec{OP}$. This line passes thru the centroid of the cube, which is the origin, and thru the midpoints of two opposite edges AB and $A'B'$. Observe that $M^2 = I$ so the magnitude of this rotation is π. This is a type 2 rotation of the cube. The permutation of vertices induced by this rotation is

$$(AB)(A'B')(CC')(DD')$$

Note that the diagonals AA' and BB' are switched whereas CC' and DD' remain fixed under M.



$(AB)(A'B')(CC')(DD')$

the permutation of vertices induced by M.