Name and Surname:
Student Number:

Math 466 - Fall 2019 - METU

Quiz 3

Consider the natural group action of S_4 on the set $X = \{1, 2, 3, 4\}$ given by $\sigma \cdot x = \sigma(x)$. For example, if $\sigma = (124)$ then $\sigma \cdot 2 = \sigma(2) = 4$.

Show that $G(1) = G(2) = \{1, 2, 3, 4\}$. Conclude that the elements 1 and 2 are in the same orbit.

$$G(1) = \{\sigma(1) \mid \sigma \in S_4\} = \{1, 2, 3, 4\}$$
$$\qquad\qquad\qquad\qquad (1)(1)\ \ (12)(1)\ \ (13)(1)\ \ (14)(1)$$

Similary $G(2) = X$. Thus 1 and 2 are in the same orbit.

Explain why G_1 equals $\{(1), (23), (24), (34), (234), (243)\}$. Determine the elements of G_2.

G_1 is the stabilizer group of 1. It consists of all elements $\sigma \in S_4$ that fixes 1.

Similarly $G_2 = \{\sigma \in S_4 \mid \sigma(2) = 2\}$
$$= \{(1), (13), (14), (34), (134), (143)\}$$

Show that G_1 and G_2 are conjugate subgroups of S_4 by finding $\sigma \in S_4$ such that $\sigma G_1 \sigma^{-1} = G_2$.

Pick $\sigma = (12) \in S_4$. It follows that
$$\sigma G_1 \sigma^{-1} = G_2$$

For example for $\tau = (234) \in G_1$, we have
$$\sigma \tau \sigma^{-1} = (12)(234)(12) = (134) \in G_2.$$