M E T U - Department of Mathematics
Math 464 - Introduction to Representation Theory

Spring 2019 Ö. Küçüksakallı	Midterm 1 March 20, 17:40 100 minutes 4 questions on 4 pages.	Surname: Name: Student No: Signature:

1	2	3	4		Total

Question 1. (25 point) For each of the following statements, determine whether it is **true** or **false**. Justify your answer briefly.

(a) The permutation module for S_3 over $\mathbb{R}$ is faithful.

(b) Let $G = C_4 = \langle a : a^4 = 1 \rangle$. There exists a nontrivial representation $\rho : G \to \mathrm{GL}(2, \mathbb{R})$ which is not faithful.

(c) If V and W are $\mathbb{F}G$-modules with $\dim(V) = \dim(W)$, then V and W are isomorphic as $\mathbb{F}G$-modules.

(d) Let W be an $\mathbb{F}G$-submodule of the $\mathbb{F}G$-module V. If $V = W \oplus U$ for some subspace U of V, then U is an $\mathbb{F}G$-submodule.

Question 2. (25 point) Let $G = C_2 = \langle a : a^2 = 1 \rangle$, and let $V = \mathbb{F}^3$ with $\mathbb{F} = \mathbb{R}$ and $\mathcal{B} = \{e_1, e_2, e_3\}$. For $(x, y, z) \in V$, define

$$1(x, y, z) = (x, y, z) \quad \text{and} \quad a(x, y, z) = (y, x, -z).$$

(a) Verify that V is an $\mathbb{F}G$-module. Describe the representation given by $\rho(g) = [g]_{\mathcal{B}}$.

(b) Decompose V as a direct sum of irreducible FG-submodules.

Question 3. (25 point) Let V be the group algebra $\mathbb{R}[G]$ with $G = S_3$.

(a) Let $x = 4(1) + (12) + (123)$ and $y = 5(12) + (13)$. Compute xy, yx and x^2.

(b) Find a **nonzero** element $z \in \mathbb{R}[G]$ such that $z((1) - (123)) = 0$.

(c) Let $w = (12) + (13) + (23)$. Show that $wr = rw$ for all $r \in \mathbb{R}[G]$.

Question 4. (25 point) Let $G = D_{12} = \langle a, b : a^6 = 1 = b^2, b^{-1}ab = a^{-1} \rangle$.

(a) You are given that the map $\rho : a^r b^s \mapsto A^r B^s$ is a group representation where

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}.$$

Is ρ is faithful? Is ρ is irreducible?

(b) You are given that the map $\sigma : a^r b^s \mapsto C^r D^s$ is a group representation where

$$C = \begin{bmatrix} \cos(\pi/3) & -\sin(\pi/3) \\ \sin(\pi/3) & \cos(\pi/3) \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Is σ is faithful? Is σ is irreducible?