Name and Surname:
Student Number:

Math 366 - Spring 2017 - METU

Quiz 4

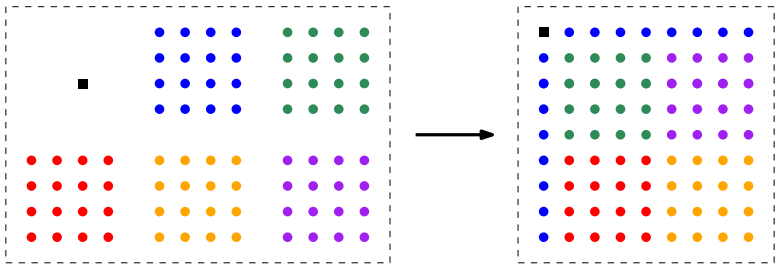
> "There are more than a hundred people living in Mettu, a market town in south-western Ethiopia. The people of Mettu (except its mayor) can form five similar squares with exactly same number of people in each square. When the mayor joins to the crowd, a single square could be arranged."

Question: Is this information enough to find the precise population of Mettu? If so, find it. Otherwise, explain why it is not enough.

Solution: This problem is equivalent with solving the Pell equation $x^2 - 5y^2 = 1$ with the restrictions

- $x^2 > 100$, and
- x^2 is the population of a town.

The fundamental solution $(9, 4)$ corresponds to the picture below.



However this solution is not suitable since $x^2 = 81 < 100$. The next two solutions are obtained by

$$\left(9 + 4\sqrt{5}\right)^2 = 161 + 72\sqrt{5}$$
$$\left(9 + 4\sqrt{5}\right)^3 = 2889 + 1292\sqrt{5}$$

Note that 2889^2 is more than a million which is too big for being the population of a town. Further solutions would have even larger x values and they are not possible, either. If we assume that **a town has population less than a million** then the population of Mettu must be $161^2 = 25921$ according to this information. We have $5 \cdot 72^2 + 1 = 161^2$.