Name and Surname:
Student Number:

Math 366 - Spring 2017 - METU

Quiz 1

Question: Choose one of the following questions to answer. Clearly indicate the question that you have chosen.

- Find the number of solutions of $25x + 19y = 4444$ where x and y are positive integers.

  Solution: Note that $25 \cdot 1 + 19 \cdot 1 = 44$. Thus $(101, 101)$ is a solution. All solutions are given by

  $$(x, y) = (101 + 19t, 101 - 25t) \qquad t \in \mathbf{Z}.$$

  The positive solutions are obtained for $-5 \leq t \leq 4$. We conclude that there are 10 such solutions

- Prove that if $n \not\equiv 2 \pmod 4$, then there is a primitive Pythagorean triple in which x or y equals n.

  Solution: There are two cases:

  - $n \equiv 1, 3 \pmod 4$: If n is odd then $n = 2k + 1$ for some integer k. Consider the Pythagorean triple $(a^2 - b^2, 2ab, a^2 + b^2)$ where $a = k + 1$ and $b = k$. Note that $n = a^2 - b^2$. Since a and b are consecutive, we have $\gcd(a, b) = 1$ and not both a and b are odd. It follows that the Pythagorean triple $(a^2 - b^2, 2ab, a^2 + b^2)$ is primitive.
  - $n \equiv 0 \pmod 4$: If $4|n$, then choose $a = n/2$ and $b = 1$. Note that $n = 2ab$. We have $\gcd(a, b) = 1$ and not both a and b are odd. Thus the Pythagorean triple $(a^2 - b^2, 2ab, a^2 + b^2)$ is primitive.