METU, Spring 2017, Math 366.

Exercise Set 5

1. In the Battle of Hastings that occurred on October 14, 1066 Harold's forces formed thirteen similar squares with exactly same number of soldiers in each square. When Harold himself joined the fray and was added to the number of his soldiers in those thirteen squares a single huge square could be arranged. How many men there must have been in Harold's force?

2. Compute the convergents of the simple continued fraction $[1; 1, 2, 1, 2, 1, 2, 1, 2]$ as real numbers. Is there a special real number to which convergents approach?

3. Find the first ten terms of the infinite continued fraction of e. Do you see a pattern?

4. For any positive integer n, find the fundamental solution of

   (a) $x^2 - (n^2 + 1)y^2 = 1$, and

   (b) $x^2 - (n^2 + 2)y^2 = 1$.

5. Find the fundamental solution of the Pell's equation $x^2 - 41y^2 = 1$.

6. For each of the following conditions, determine if there are infinitely many primitive Pythagorean triples $a^2 + b^2 = c^2$ satisfying that property.

   - $c - b = 1$
   - $b - a = 1$
   - $c - b = 2$
   - $b - a = 2$
   - $c - b = 3$
   - $b - a = 3$

7. If d is divisible by a prime $p \equiv 3 \pmod 4$, show that the negative Pell's equation $x^2 - dy^2 = -1$ has no solution.

8. Give an infinite set of solutions of the Diophantine equation $x^2 - 6y^2 = 19$.

9. Show that the Diophantine equation $x^2 + y^2 = 2z^2$ has infinitely many solutions in positive integers with $x = 1$. Describe a process by using which you can find all such solutions.