Name and Surname:
Student Number:

Math 366 - Spring 2015 - METU

Quiz 3

Question 1: Let α and β be Gaussian integers. Prove or disprove each of the following statements:

- If $\gcd(\alpha, \beta) = 1$, then $\gcd(N(\alpha), N(\beta)) = 1$.

  Solution: Consider $\alpha = 2 + i$ and $\beta = 2 - i$. These are distinct Gaussian primes so we have $\gcd(\alpha, \beta) = 1$. However $\gcd(N(\alpha), N(\beta)) = 1$ does not hold.

- If $\gcd(N(\alpha), N(\beta)) = 1$, then $\gcd(\alpha, \beta) = 1$.

  Solution: We prove the statement by its contrapositive. Suppose that $\gcd(\alpha, \beta) \neq 1$. Then there exist a Gaussian integer γ such that γ divides both α and β and $N(\gamma) > 1$. It follows that $N(\gamma)$ divides $N(\alpha)$ and $N(\beta)$. We conclude that $\gcd(N(\alpha), N(\beta)) \neq 1$.