METU, Spring 2015, Math 366.

Exercise Set 1

1. Show that the Diophantine equation $ax + by + cz = d$ has a solution if and only if $\gcd(a, b, c)$ divides d.
2. Find all integer solutions of the following equations:
   (a) $2x + 3y + 4z = 5$.
   (b) $30x + 42y + 70z + 105t = 1$.
3. Find all integer solutions of the system of equations $3x + 5y = 1$ and $7x + 11y = 1$.
4. Find all solutions of the following Diophantine equations in two different ways, the basic approach and the geometric approach.
   (a) $x^2 + 3y^2 = z^2$.
   (b) $x^2 + y^2 = 5z^2$.
5. Let $n \geq 3$ be given. Show that there is Pythagorean triple (x, y, z) such that one of x, y, z is n.
6. Find all integer solutions of the system of equations $y + z = 1$ and $x^2 + y^2 = z^2$.
7. Find a Pythagorean triple (x, y, z) such that $x + y + z = 366$.
8. For which values of m, is the Diophantine equation $x^2 - y^2 = m$ solvable? Show that the equation $x^2 - y^2 = m^3$ is solvable for any m.