Name and Surname:
Student Number:

Math 366 - Spring 2013 - METU

Quiz 4

(1) Let α, β be non-zero Gaussian integers. If α divides β, then show that $N(\alpha)$ divides $N(\beta)$. Does the converse hold? (Hint: Find Gaussian integers α and β such that $N(\gcd(\alpha, \beta)) \neq \gcd(N(\alpha), N(\beta))$.)

Solution: If α divides β then there exists a Gaussian integer γ such that $\beta = \alpha\gamma$. Comparing the norms of both sides, we see that $N(\beta) = N(\alpha)N(\gamma)$. We conclude that $N(\alpha)$ divides $N(\beta)$. The converse is not true. Pick $\alpha = 2 + i$ and $\beta = 2 - i$. Then $N(\alpha)$ divides $N(\beta)$ since they are both equal to five. On the other hand α does not divide β since $\beta/\alpha = 3/5 - 4/5i$.

(2) Let α, β be non-zero Gaussian integers. Show that $N(\gcd(\alpha, \beta)) | \gcd(N(\alpha), N(\beta))$.

Solution: Observe that $N(\gcd(\alpha, \beta))$ divides $N(\alpha)$ and $N(\beta)$ by the previous question. Therefore $N(\gamma)$ divides the greatest common divisor of $N(\alpha)$ and $N(\beta)$.

(3) Find the greatest common divisor of Gaussian integers $\alpha = 17 + i$ and $\beta = 10$. Use the inverse division algorithm in order to determine Gaussian integers η and λ such that $\gcd(\alpha, \beta) = \alpha\eta + \beta\lambda$.

Solution: Let us apply the division algorithm to find the greatest common divisor of α and β.

$$\begin{aligned} \alpha &= \beta \cdot 1 + (7 + i) \\ \beta &= (7 + i) \cdot 1 + (3 - i) \\ 7 + i &= (3 - i) \cdot (2 + i) + 0 \end{aligned}$$

Thus $\gcd(\alpha, \beta) = 3 - i$. Now we find η and λ.

$$\begin{aligned} \gcd(\alpha, \beta) &= \beta - (7 + i) \\ &= \beta - (\alpha - \beta) \\ &= (-1)\alpha + 2\beta. \end{aligned}$$

We can pick $\eta = -1$ and $\lambda = 2$.