M E T U
Department of Mathematics

Elementary Number Theory I Midterm 2	
Code : *Math 365* Acad. Year : *2017* Semester : *Fall* Instructor : *Küçüksakallı* Date : *December 4, 2017* Time : *17:40* Duration : *120 minutes*	Last Name : Name : Student No. : Signature :
	8 QUESTIONS ON 4 PAGES 100 TOTAL POINTS

1	2	3	4	5	6	7	8		

1. (15pts) Find the remainder of $N = 20! + 2^{20}$ upon divison by 23.

2. (10pts) Show that $a^{365} \equiv a \pmod{29}$ for all integers a.

3. (15pts) Prove that $\tau(n)$ is an odd integer if and only if n is a perfect square.

4. (10pts) Find all n, if there is any, such that $n!$ has precisely 60 digits of zeros at the end in its decimal expression.

5. (15pts) Define $F(n) = \sum_{d|n} \mu(d)\sigma(d)$. Compute $F(10!)$.

6. (10pts) Find a function $f(n)$ such that $\sum_{d|n} f(d) = n^2 + 1$. Compute $f(6)$ and $f(12)$.

7. (15pts) Find all solutions of the equation $\phi(n) = 16$.

8. (10pts) Let k be a fixed positive integer. Show that the equation $\phi(n) = k$ has only a finite number of solutions. (Hint: Show that $\phi(n) > \sqrt{n}/2$.)