

M E T U
Department of Mathematics

| Elementary Number Theory I | | | | | | | |
|----------------------------------|---|---|---|--|---|---|---|
| Midterm 1 | | | | | | | |
| Code : <i>Math 365</i> | | | | Last Name : | | | |
| Acad. Year : <i>2017</i> | | | | Name : | | | |
| Semester : <i>Fall</i> | | | | Student No. : | | | |
| Instructor : <i>Küçükşakallı</i> | | | | Signature : | | | |
| Date : <i>November 6, 2014</i> | | | | 8 QUESTIONS ON 4 PAGES 100 TOTAL POINTS | | | |
| Time : <i>17:40</i> | | | | | | | |
| Duration : <i>120 minutes</i> | | | | | | | |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

1. (15pts) Let $a = 3655$ and $b = 2021$. Show that $\gcd(a, b) = 43$. Find $x, y \in \mathbb{Z}$ such that $ax + by = 43$.

2. (10pts) Give the precise statement of the Fundamental Theorem of Arithmetic. Provide a few examples to illustrate its conclusion.

3. (15pts) Let a, b and c be positive integers and let x and y be integer variables. Prove or disprove the following statement: “The Diophantine equation $ax + by = c$ has a solution if and only if the Diophantine equation $ax + cy = b$ has a solution.”

4. (10pts) Let a and b positive integers such that $\gcd(a, b) = 1$. Suppose that ab is a perfect square, i.e. $ab = c^2$ for some integer c . Show that each one of the integers a and b is a perfect square.

5. (15pts) Let $S = \{2, 3, 5, 7, 13, 17, 19, 23, 29, 37, 43, \dots\}$ be the set of primes that are NOT of the form $5k + 1$. Show that S is infinite. (Do not use Dirichlet's Theorem.)

6. (10pts) Consider the 1000 digit number $N = 111 \dots 111$ which consists of 1000 digits of ones. Determine the remainder of N upon division by 13.

7. (15pts) Find all solutions of the following system of equations:

$$2x \equiv 2 \pmod{4}, \quad 3x \equiv 1 \pmod{5}, \quad 4x \equiv 3 \pmod{9}.$$

8. (10pts) The student ID-number of a university is of the form $a_1a_2a_3a_4a_5a_6 - c$ where six digits are followed by a check digit c that satisfies the congruence $c \equiv a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 + 6a_6 \pmod{10}$. Cahit's student ID-number is $365x42 - 8$. Is it possible to find the missing digit x ? If it is possible, then find it. If it is not possible, then explain why.