M E T U - Department of Mathematics

Math 262 - Linear Algebra II

Spring 2019 Ö. Küçüksakallı	Midterm 2 April 17, 17:40 100 minutes 5 questions on 4 pages.	Surname: Name: Student No: Signature:

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Question 1. (25 point) For each of the following statements, determine whether it is **true** or **false**. Justify your answer briefly.

(a) An operator T and its adjoint T^* have the same eigenvectors.

(b) Let T be an invertible linear operator on a finite dimensional inner product space. Then T^* is invertible and $(T^*)^{-1} = (T^{-1})^*$.

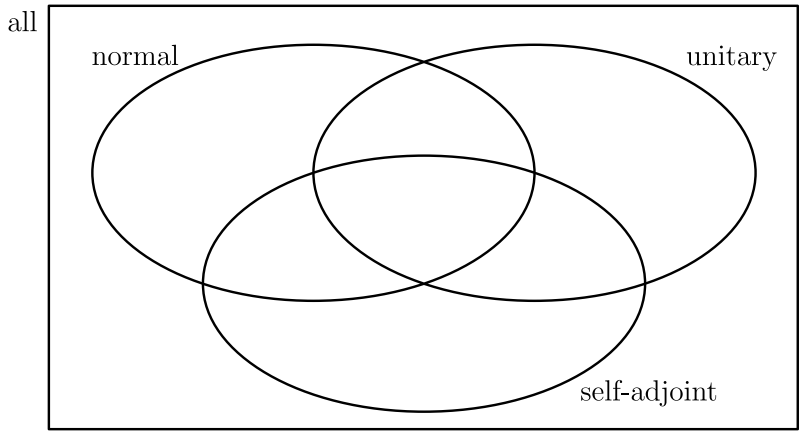
(c) Let T be an orthogonal operator on a finite dimensional real inner product space. Then T is diagonalizable.

(d) Let $V = \mathbb{R}^2$ with the standard inner product. The orthogonal projection from V onto $W = \text{span}(\{(1,1)\})$ is given by $T(a,b) = (a+b, a+b)$ for all $(a,b) \in V$.

Question 2. (15 point) Let $V = \mathbb{C}^2$ be the complex vector space with the standard inner product. Recall that $L_M : V \to V$ is given by $L_M(x) = Mx$. Set

$$A = \begin{bmatrix} i & i \\ i & i \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & i \\ i & 0 \end{bmatrix}, D = \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}, E = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Place the operators L_A, L_B, L_C, L_D, L_E in the correct region of the Venn diagram below.



Question 3. (10 point) Let $V = P_1(\mathbb{R})$ with $\langle f, g\rangle = \int_0^2 f(t)g(t)dt$. Let $T : V \to V$ be the linear operator defined by $T(a + bx) = b$. Determine $T^*(c + dx)$.

Question 4. (25 point) Let V be a finite dimensional inner product space, and let W be a proper subspace of V.

(a) State the definition of $W^\perp$. Prove that $W^\perp$ is a subspace of V.

(b) Let $x \in V \setminus W$. Prove that there exists $y \in W^\perp$ such that $\langle x, y \rangle \neq 0$.

(c) Show that $(W^\perp)^\perp = W$.

Question 5. (25 point) For the matrix A below, find an orthogonal matrix Q such that $D = Q^t AQ$ is a diagonal matrix. You are given that $\det(A - tI) = -(t-2)^2(t-5)$.

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}.$$