M E T U - Department of Mathematics
Math 262 - Linear Algebra II

Spring 2019 Ö. Küçüksakallı	Midterm 1 March 19, 17:40 100 minutes 4 questions on 4 pages.	Surname: Name: Student No: Signature:

1	2	3	4		Total

Question 1. (25 point) For each of the following statements, determine whether it is **true** or **false**. Justify your answer briefly.

(a) Let T be a linear map. The scalar zero is never an eigenvalue of T.

(b) Let V be a finite dimensional vector space and let W_1 be any subspace of V. Then there exists a subspace W_2 of V such that $V = W_1 \oplus W_2$.

(c) Let $A \in M_{n\times n}(\mathbb{R})$ be a square matrix. Then the dimension of $\operatorname{span}(\{I_n, A, A^2, \ldots\})$ is less than or equal to n.

(d) The formula $\langle A, B\rangle = \operatorname{tr}(A + B)$ defines an inner product on $M_{2\times 2}(\mathbb{R})$.

Question 2. **(25 point)** Let $\beta = \{1, x, x^2\}$ be the standard ordered basis for $P_2(\mathbb{R})$. Let T be the linear operator on $P_2(\mathbb{R})$ defined by $T(f(x)) = 2f'(x) + f(1)x$.

(a) Find $[T]_\beta$.

(b) Find the characteristic polynomial of T.

(c) Find a basis for each eigenspace.

(d) Find a basis α of V such that $[T]_\alpha$ is a diagonal matrix.

Question 3. **(25 point)** Let $V = M_{2\times 2}(\mathbb{R})$. Consider the linear map $T : V \to V$ given by the formula

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} 3b & 2a+b \\ 4d-c & 5c \end{bmatrix}.$$

(a) Let W be the T-cyclic space generated by the vector $E^{11} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. Find a basis for the subspace W.

(b) Find the characteristic polynomial of the restricted map $T|_W$.

(c) Let U be the T-cyclic space generated by the vector $E^{22} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. Find a basis for the subspace U.

(d) Find the characteristic polynomial of the restricted map $T|_U$.

(e) Explain briefly why V is the direct sum of W and U. Find the characteristic polynomial of T **by using the parts (b) and (d)**.

Question 4. (25 point) Let $V = R^3$ be the inner product space with the standard inner product.

(a) Apply the Gram-Schmidt process to $S = \{(1,2,2), (1,0,0), (0,1,0)\}$.

(b) Find an orthonormal basis β of V that contains $(1/3, 2/3, 2/3)$.

(c) Compute the Fourier coefficients of $w = (2,6,2)$ relative to β. Express w as a linear combination of vectors in β.