M E T U - Department of Mathematics
Math 262 - Linear Algebra II

Spring 2019 Ö. Küçüksakallı	Final May 24, 13:30 120 minutes 4 questions on 4 pages.	Surname: Name: Student No: Signature:

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Question 1. (25 point) For each of the following statements, determine whether it is **true** or **false**. Justify your answer briefly.

(a) There exists a matrix $A \in M_{3\times 3}(\mathbb{C})$ such that $A^3 = I$ but $A^2 \neq I$.

(b) A linear operator is diagonalizable if and only if its minimal polynomial splits.

(c) Every orthonormal set is linearly independent.

(d) A generalized eigenvector of T is always an eigenvector of T.

Question 2. (25 point) Let $S = \{e^x, xe^x, x^2e^x, x^3e^x\}$ be a subset of real valued functions. Define $V = \text{span}(S)$. Let $T : V \to V$ be the linear operator given by $T(f) = f'$.

(a) Find the minimal polynomial of T.

(b) Find a Jordan canonical basis β for V. Determine $[T]_\beta$.

(c) Find a rational canonical basis γ for V. Determine $[T]_\gamma$.

Question 3. (25 point) Let $V = P_2(\mathbb{R})$. Define $H(f, g) = \int_{-2}^{2} f(t)g(t)dt$.

(a) Is H a bilinear form on V? Explain briefly.

(b) Let $\beta = \{1, x, x^2\}$ be the standard basis for V. Find $\psi_\beta(H)$.

(c) Find a basis γ for V such that $\psi_\gamma(H)$ is a diagonal matrix.

Question 4. (25 point) Let $A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

(a) Find an invertible matrix Q such that $Q^{-1}AQ = B$

(b) The direct approach requires 2018 matrix multiplications to compute A^{2019}. Use the previous part, to express A^{2019} as a product of at most 3 matrices.