M E T U
Department of Mathematics

Calculus for Mathematics Students I Midterm 2	
Code : *Math 153* Acad. Year : *2016* Semester : *Spring* Instructor : *Küçüksakallı* Date : *April 27, 2016* Time : *17:40* Duration : *120 minutes*	Last Name : Name : Student No. : Signature :
	4 QUESTIONS ON 4 PAGES 100 TOTAL POINTS

1	2	3	4	5	6	7		

1. (2x6=12pts) Evaluate the following limits if they exist. (Do not use L'Hospital's Rule.)

- $\lim_{x \to \frac{1}{2}} \dfrac{\cos(\pi x)}{x - \frac{1}{2}}$

- $\lim_{x \to 1} \dfrac{x^{2016} + x^{153} - 2}{x - 1}$

2. (2x6=12pts) Evaluate the following derivatives. (Do not simplify.)

- $\dfrac{d}{dx}\left[x^3 \tan(\cos(x^2))\right]$

- $\dfrac{d}{dx}\left(\dfrac{x}{x^{153} + \frac{2016}{x^2+1}}\right)$

3. (12pts) Find the derivative of $f(x) = \sqrt[3]{x}$ by using the definition.

4. (14pts) Use a suitable linearization to approximate $\sqrt[3]{128}$. Estimate the size of the error.

5. (5x5=25pts) Determine if the given statement is true or false. If it is true, prove it. If it is false, give a counterexample.

- Suppose that $f(x)$ is continuous on $[0, 2]$ and $f(0) = f(2)$. Then there exists a number $c \in [0, 1]$ such that $f(c) = f(c + 1)$.

- Suppose that f is a differentiable function on the whole real line such that $f(0) = 0$ and $f(1) = 1$. Then there exists a number $c \in (0, 1)$ such that $f'(c) = 2c$.

- There exists a function $f(x)$ such that $f'(x) = |x|$.

- If $f(x)$ is differentiable at $x = x_0$, then $|f(x)|$ is differentiable at $x = x_0$.

- If $|f(x)|$ is differentiable at $x = x_0$, then $f(x)$ is differentiable at $x = x_0$.

6. (10pts) Find an equation of the tangent line to the curve defined by the equation $x^2 - y^2 = \sin(y)$ at the point (π, π).

7. (15pts) Let $g(x) = \begin{cases} ax + a & \text{if } x < a, \\ 2a + 2 & \text{if } x = a, \\ bx - 1 & \text{if } x > a. \end{cases}$

a) Determine all possible values of a and b so that $g(x)$ is continuous at $x = a$.

b) Determine all possible values of a and b so that $g(x)$ is differentiable at $x = a$.