M E T U
Department of Mathematics

Calculus for Mathematics Students I
Midterm 1

Code : *Math 153* Acad. Year : *2016* Semester : *Spring* Instructor : *Küçüksakallı* Date : *March 30, 2016* Time : *17:40* Duration : *120 minutes*	Last Name : Name : Student No. : Signature : 4 QUESTIONS ON 4 PAGES 100 TOTAL POINTS

1	2	3	4		

1. (4x6=24pts) Evaluate the following limits if they exist. (Do not use L'Hospital's Rule.)

- $\lim_{x\to 2} \dfrac{\sin(\pi x)}{x-2}$

- $\lim_{x\to 1} \dfrac{\sqrt[3]{x}-1}{\sqrt{x}-1}$

- $\lim_{x\to -2^+} \dfrac{|x|-2}{|x+2|}$

- $\lim_{x\to \infty} \dfrac{153+\cos(x)}{x+2016}$

2. (3+10+3+10=26pts)

- State the $\varepsilon - \delta$ definition of $\lim_{x \to a} f(x) = L$.

- Show that $\lim_{x \to 2}(3x - 5) = 1$ by using the $\varepsilon - \delta$ definition.

- State the formal definition of $\lim_{x \to a^+} f(x) = \infty$.

- Show that $\lim_{x \to 1^+} \dfrac{3}{x - 1} = \infty$ by using the formal definition.

3. (5x5=25pts) Determine if the given statement is true or false. If it is true, prove it. If it is false, give a counterexample.

- If $f(x)$ and $g(x)$ are increasing functions on $(0, \infty)$, then $f(x)g(x)$ is an increasing function on $(0, \infty)$.

- If $f(x)$ is an even function and $\lim_{x \to 0^+} f(x) = 153$, then $\lim_{x \to 0} f(x) = 153$.

- If $\lim_{x \to a}[f(x) + g(x)]$ exists, then $\lim_{x \to a}[f(x) - g(x)]$ exists.

- If $\lim_{x \to a} |f(x)| = 0$, then $\lim_{x \to a} f(x) = 0$.

- If $\lim_{x \to \infty} \dfrac{1}{f(x)} = 0$, then there exists $r \in \mathbb{R}$ such that $f(x) > 153$ whenever $x > r$.

4. (5+5+5+10=25pts) Consider the function $f(x) = \begin{cases} x & \text{if } x < 0, \\ 2x & \text{if } x \geq 0. \end{cases}$ from $\mathbb{R}$ to $\mathbb{R}$.

- Is $f(x)$ one-to-one and onto? Sketch the graph of $f^{-1}(x)$.

- Sketch the graph of $y = 2 - f(x+1)$.

- Sketch the graph of $y = f(x^2)$.

- Find an odd function $o(x)$ and an even function $e(x)$ such that $f(x) = o(x) + e(x)$. Sketch the graphs of $o(x)$ and $e(x)$.