M E T U
Department of Mathematics

Calculus for Mathematics Students I
Final

Code : *Math 153* Acad. Year : *2016* Semester : *Spring* Instructor : *Küçüksakallı* Date : *June 4, 2016* Time : *9:30* Duration : *135 minutes*	Last Name : Name : Student No. : Signature : 7 QUESTIONS ON 5 PAGES 100 TOTAL POINTS

1	2	3	4	5	6	7		

1. (2x5=10pts) Evaluate the following limits if they exist.

- $\lim_{x\to\infty}\left(1+\frac{153}{x}\right)^{x}$

- $\lim_{x\to 1}\left(\frac{1}{\ln(x)}-\frac{1}{x-1}\right)$

2. (2x5=10pts) Evaluate the following derivatives.

- $\frac{d}{dx}\left[\ln(\arctan(x^2+1))\right]$

- $\frac{d}{dx}\left[\sin(x)^x\right]$

3. (10pts) Show that $\lim_{x\to 7}(3x^2 + x - 1) = 153$ by using the $\varepsilon - \delta$ definition.

4. (25pts) Sketch the graph of $f(x) = \frac{x^2-4}{x^2+1}$ step by step as follows:

- Find all x and y intercepts of f.

- Find all asymptotes of f.

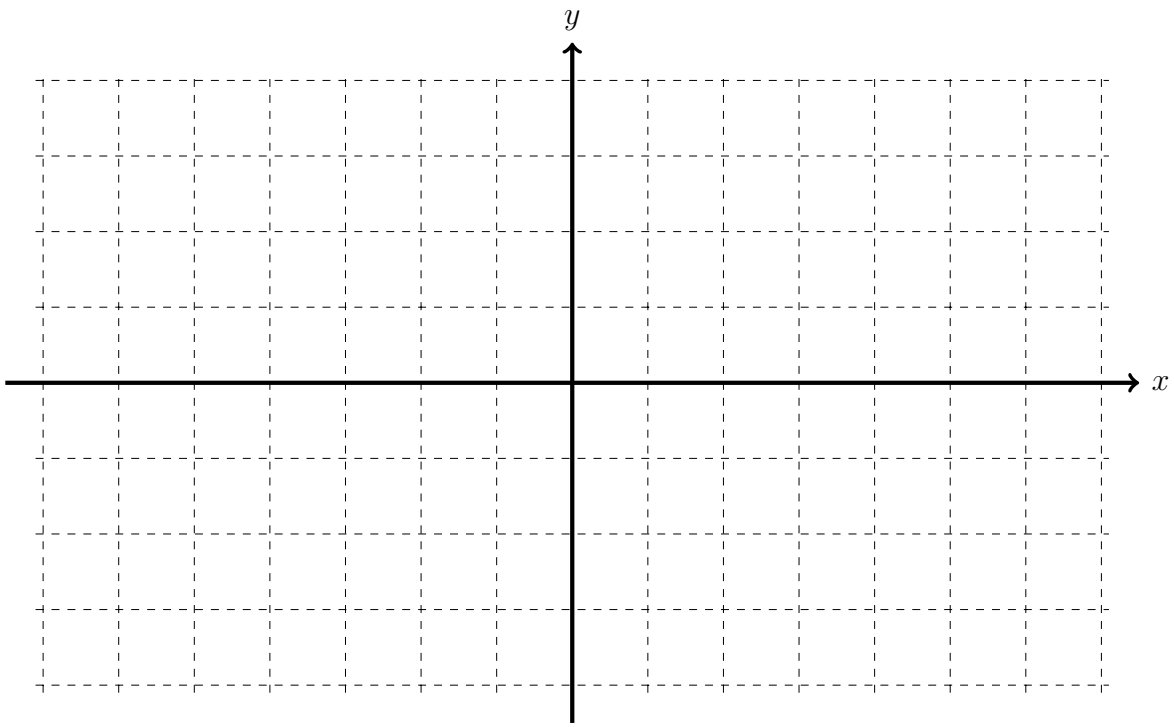
- Determine the intervals on which $f(x)$ is increasing and on which $f(x)$ is decreasing.

- Find the point(s), if any, at which $y = f(x)$ has local maximum and local minimum.

- Determine the intervals on which $y = f(x)$ is concave up and on which $y = f(x)$ is concave down.

- Find the inflection point(s), if any, of the graph $y = f(x)$.

- Sketch the graph of the function $y = f(x)$ on the given coordinate plane.



5. (10pts) Find the points on the ellipse $x^2 + 4y^2 = 4$ that are closest to the point $(1, 0)$.

6. (10pts) A ladder 5 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 1 m/s, how fast is the angle between the top of the ladder and the wall changing when the angle is $\pi/4$ radians.

7. (5x5=25pts) Determine if the given statement is true or false. Give reasons for your answer.

- If $f : \mathbb{R} \to \mathbb{R}$ is a twice differentiable function such that $f''(153) = 0$, then the graph $y = f(x)$ has an inflecton point at $x = 153$.

- The derivative $f'(0)$ does not exist for the function $f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$

- $\left(\arcsin(\frac{1}{2}) - \arcsin(\frac{1}{3})\right) \leq \frac{1}{3\sqrt{3}}$.

- If $f(x)$ is continuous on the interval $(0, 153]$ then it has either an absolute maximum or an absolute minimum on that interval.

- If $y = f(x)$ is differentiable on $\mathbb{R}$ and $\lim_{x\to\infty} f'(x) = 0$, then $y = f(x)$ has a horizontal asymptote.