# M E T U <br> Department of Mathematics 

|  | Discrete Mathematics Makeup |  |  |  |
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| Code <br> Acad. Year <br> Semester <br> Instructor | : Math 112 <br> : 2010-2011 <br> : Spring <br> : Bhupal, Küçüksakall, Okutmuştur, Seven. | Last Name <br> Name <br> Department <br> Signature | Student No : |  |
| Date <br> Time <br> Duration | $\begin{aligned} & : 06.06 .2011 \\ & : 9.30 \\ & : 120 \text { minutes } \end{aligned}$ | 7 Questions on 4 Pages Total 80 Points |  |  |
| ${ }^{2}$ | $\left.\left.{ }^{3} \quad\right\|^{4} \quad\right\|^{5}$ | ${ }^{7}$ |  |  |

1. $(6+6=12$ pts. $)$ For each pair of graphs shown below, determine if they are isomorphic or not. Explain your answer.
a)

b)

2. (10 pts.) There are 28 girls and 21 boys in a classroom. The number of girls with black hair is 8 and the number of boys with black hair is 9 . If randomly chosen two children have both black hairs, what is the conditional probability that they are both girls?
3. (12 pts.) In how many ways can eight different jobs are assigned to five employees so that each employee is assigned at least one job? Justify your answer.
4. $\left(6+6=12 \mathrm{pts}\right.$.) Let $a_{n}$ be the number of strings of length $n$ consisting of letters $A, B, C$ that do not contain three consecutive $A$ 's. (For example if $n=5$, then $A A B C A$ is acceptable but BAAAC is not acceptable.)
a) Compute $a_{1}, a_{2}$ and $a_{3}$.
b) Find a recurrence relation for $a_{n}$. (Do not solve it.)
5. (10 pts.) If 11 integers are selected from $\{1,2,3, \ldots, 100\}$, prove that there are at least two of them, say $x$ and $y$, such that $0<|\sqrt{x}-\sqrt{y}|<1$.
6. $(6+6=12$ pts. $)$
a) If a connected planar simple graph has $e$ edges and $v$ vertices with $v \geq 3$ and no circuits of length three, then show that $e \leq 2 v-4$.
b) Use part a) to show that $K_{3,3}$ is not planar.
7. $(6+6=12$ pts. $)$ Construct two non-isomorphic graphs with 8 vertices, each of degree 4 .
