# M ET U <br> Department of Mathematics 

|  | Discrete Mathematics Final |  |  |  |
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| Code $:$ Math 112 <br> Acad. Year $:$ 2010-2011 <br> Semester $:$ Spring <br> Instructor $:$ Bhupal, Kücüksakall, <br>  Okutmuştur, Seven. <br> Date $:$ 01.06.2011 <br> Time $: 9.30$ <br> Duration $: 120$ minutes |  | Last Name <br> Name <br> Department <br> Signature | Student No |  |
|  |  | 7 Questions on 4 Pages Total 80 Points |  |  |
| ${ }^{2}$ | $\left.\left.{ }^{3} \quad\right\|^{4} \quad\right\|^{5}$ | 7 |  |  |

1. $(6+6=12$ pts. $)$ For each pair of graphs shown below, determine if they are isomorphic or not. Explain your answer.
a)

b)

2. ( $\mathbf{1 0}$ pts.) The probability that Ayşe hits a target is $1 / 4$ and the probability that Bora hits the same target is $1 / 8$. If each fires once and the the target is hit exactly once, what is the conditional probability that Bora hits the target?
3. (12 pts.) Solve the recurrence relation

$$
a_{n}=-4 a_{n-1}-4 a_{n-2}, \quad n \geq 2
$$

with initial conditions $a_{0}=1$ and $a_{1}=2$.
4. $(\mathbf{6}+\mathbf{6}=12$ pts. $)$ In how many ways can 10 distinct books be distributed among 14 children if a) no student gets more than one book?
b) five students get two books each?
5. (10 pts.) Let $S=\left\{\left(x_{i}, y_{i}\right): 1 \leq i \leq 5\right\}$ be a set of five points with integer coordinates in the $x y$-plane. Show that there is a pair of points in $S$ such that the midpoint of the line connecting this pair has integer coordinates.
6. $(6+6=12$ pts. $)$ For which values of $m$ and $n$, the complete bipartite graph $K_{m, n}$ has
a) an Euler circuit?
b) a Hamilton circuit?
7. $(\mathbf{6}+\mathbf{6}=\mathbf{1 2} \mathbf{p t s . )}$ A planar graph $G$ has 10 vertices each of degree 4 .
a) How many edges does the graph $G$ have?
b) How many regions are there in a planar representation of $G$ ?

