METU, Fall 2010, Math 111, Section 1.

Homework 5

1. In each case, determine whether or not R is a partial order. If so, is it a total order?
   - $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x \geq y\}$.
   - $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 \geq y^2\}$.
   - $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} : |x| > |y|\}$.

2. Suppose R is a relation on A. Prove that R is both symmetric and antisymmetric if and only if $R \subseteq \mathrm{id}_A$.

3. Suppose R_1 and R_2 are partial orders on A. For each part, give either a proof or a counterexample to justify your answer.
   - Must $R_1 \cap R_2$ be a partial order on A?
   - Must $R_1 \cup R_2$ be a partial order on A?

4. Let D be the divisibility relation on the set of integers. Let $B = \{x \in \mathbb{Z} : x > 1\}$.
   - What are the D-minimal elements of B?
   - Does B have a D-minimum element? If so, what is it?

5. Prove that for all $n \in \mathbb{N}$,
   - $1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$.
   - $a - b$ divides $a^n - b^n$ where $a, b \in \mathbb{Z}$.