Homework 3

1. For $A, B, C \subseteq U$, prove that

$$A \times (B \setminus C) = (A \times B) \setminus (A \times C).$$

2. Suppose that $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$. Consider the relations $R = \{(1, 4), (1, 6), (2, 6), (3, 5)\}$ and $S = \{(4, 5), (4, 6), (6, 4), (5, 5)\}$. Note that R is a relation from A to B and S is a relation from B to B. Determine the following relations:

   - $S \circ R$
   - $S \circ S^{-1}$

3. Suppose R is a relation from A to B and S is a relation from B to C. Prove that $S \circ R = \varnothing$ if and only if $\mathrm{Ran}(R)$ and $\mathrm{Dom}(S)$ are disjoint.

4. For each of the following functions, determine whether it is injective and determine its range:

   - $f : \mathbb{Z} \to \mathbb{Z}, \ f(x) = 2x + 1$.
   - $f : \mathbb{Q} \to \mathbb{Q}, \ f(x) = 2x + 1$.
   - $f : \mathbb{R} \to \mathbb{R}, \ f(x) = 2^x + 1$.
   - $f : [0, \pi/2] \to \mathbb{R}, \ f(x) = \sin(2x)$.
   - $f : [0, \pi/2] \to \mathbb{R}, \ f(x) = \cos(2x)$.

5. Show that composition of two functions is a function.