METU, Fall 2010, Math 111, Section 1.

Homework 2

1. Simplify the logical formula

$$\neg(((P \vee Q) \wedge R) \Rightarrow \neg Q)$$

   by writing it with symbols $\{\neg, \wedge\}$ only.

2. Using the system of detachment, demonstrate the validity of the argument

$$\{P \Rightarrow R, \neg P \Rightarrow Q, Q \Rightarrow S\} \vDash \neg R \Rightarrow S.$$

3. Show that

$$\{P \Rightarrow Q, \neg Q\} \vDash \neg P$$

   is valid by using truth values and tables. Using this argument, illustrate the following rules of logical reasoning:

   - Contradiction
   - Contraposition
   - Deduction

4. For the universe of integers, let $P(x, y)$ mean x divides y. State whether the following statements are true or false. Justify briefly.

   - $\exists x \forall y P(x, y)$
   - $\exists y \forall x P(x, y)$
   - $\exists x \exists y P(x, y)$
   - $\forall x \forall y P(x, y)$

5. Express the given sentences in logical symbols where the universe of discourse is positive integers. Write the negation of these sentences in logical symbols and words.

   - If x is odd then $x^2 - 1$ is even.
   - If x is a prime number and $x \equiv 1 \pmod 4$ then there exists m and n such that $x = m^2 + n^2$.