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Name and Surname:
Student number:

METU MATH 111, EXAM 1

Tuesday, November 9, 2010, at 17:40

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Instructions: There are 7 numbered problems on 4 pages. It should be obvious to the grader how to read your solutions. Please work carefully.

Problem 1. Complete the following 'full' truth-table. (You should write the possible values of the variables in the standard order. The symbol & has the same meaning as $\wedge$.)

$(P$	$\vee$	$\neg$	$Q)$	$\Rightarrow$	$(R$	&	$Q)$
0	1	1	0	0	0	0	0
1	1	1	0	0	0	0	0
0	0	0	1	1	0	0	1
1	1	0	1	0	0	0	1
0	1	1	0	0	1	0	0
1	1	1	0	0	1	0	0
0	0	0	1	1	1	1	1
1	1	0	1	1	1	1	1

Problem 2. Find a formula F with the following truth-table. Explain briefly what you did.

P	Q	R	F
0	0	0	1
1	0	0	1
0	1	0	0
1	1	0	0
0	0	1	0
1	0	1	0
0	1	1	1
1	1	1	0

Disjunctive Normal Form:

$$(\neg P\wedge\neg Q\wedge\neg R)\vee(P\wedge\neg Q\wedge\neg R)\vee(\neg P\wedge Q\wedge R)$$

Problem 3. Write down the *negation* of the following sentence, without using the negating connective $\neg$ (you may use the sign of inequality $\neq$).

$$\forall x \, \forall y \, (x^2 = y^2 \implies x = y \vee x = -y).$$

$$\exists x \, \exists y \, (x^2 = y^2 \wedge x \neq y \wedge x \neq -y)$$

Problem 4. Establish the following logical entailment by the method of your choice. (You may for example write a formal proof in the 'System of Detachment', or do something else, as long as it is clear to the reader what you are doing. The symbol & has the same meaning as $\wedge$.)

$$\boldsymbol{P}, \, \neg \boldsymbol{P} \vee \boldsymbol{Q}, \, \boldsymbol{Q} \Rightarrow \boldsymbol{R}, \, \neg(\boldsymbol{R} \,\&\, \neg \boldsymbol{S}) \models \boldsymbol{S}.$$

	Steps	Reasons
1)	$\neg P \vee Q$	Premise
2)	$P \Rightarrow Q$	Defn of $\Rightarrow$
3)	$Q \Rightarrow R$	Premise
4)	$P \Rightarrow R$	Hyp. Syl.
5)	P	Premise
6)	R	Detachment
7)	$\neg(R \wedge \neg S)$	Premise
8)	$\neg R \vee S$	De Morgan
9)	$R \Rightarrow S$	Defn of $\Rightarrow$
10)	S	Detachment

Problem 5. Assume that the universe consists of the integers (that is, all variables stand for integers); let $E(x)$ stand for the property of x being even; and let $P(x)$ stand for x being prime. Using the quantifiers $\forall$ and $\exists$ as needed, write down symbolically the following statement:

Every even integer that is greater than 4 is the sum of two primes.

$$\forall x \left[E(x) \wedge x > 4 \Rightarrow \exists a \exists b \left(P(a) \wedge P(b) \wedge x = a+b \right) \right]$$

Problem 6. Assuming A and B are arbitrary sets, prove

$$A \not\subseteq B \iff A \cap B^c \neq \varnothing.$$

$\Rightarrow$) Suppose that A is not a subset of B. Then there exists an element $x \in A$ which is not in B. In other words $x \in A$ and $x \in B^c$. This means that $x \in A \cap B^c$ and therefore this intersection is not empty.

$\Leftarrow$) Conversely suppose that $A \cap B^c$ is not empty. Hence we can pick an element x in this intersection. Such an element satisfies $x \in A$ and $x \in B^c$. This follows that A is not a subset of B. This finishes the proof.

Problem 7. Explain what is wrong with the following 'proof' of the claim that, for any sets A, B, C, and D,

$$(A \cup B) \cap (C \cup D) \subseteq (A \cap C) \cup (B \cap D). \qquad (*)$$

(Parts of the proof are numbered for ease of reference.)

(a) Suppose $x \in (A \cup B) \cap (C \cup D)$;
(b) we shall show $x \in (A \cap C) \cup (B \cap D)$.
(c) We have $x \in A \cup B$
(d) and $x \in C \cup D$.
(e) Therefore $x \in A$ or $x \in B$;
(f) also, $x \in C$ or $x \in D$.
(g) If $x \in A$ and $x \in C$, then $x \in A \cap C$.
(h) If $x \in B$ and $x \in D$, then $x \in B \cap D$.
(i) Since $A \cap C \subseteq (A \cap C) \cup (B \cap D)$
(j) and $B \cap D \subseteq (A \cap C) \cup (B \cap D)$,
(k) it follows that $x \in (A \cap C) \cup (B \cap D)$.
(l) Therefore $(*)$ holds.

The line (k) does not follow from the previous lines.