

Modeling with Differential Equations

1. We have a tank of 100 *gal* of water, which contains Q_0 amount of salt dissolved. Assume that water is entering into the tank at a rate of r *gal/min* and the well-stirred mixture leaves the tank with the same rate. The entering water contains $1/4$ *lb* of salt per gallon. Find the amount of salt $Q(t)$ in the tank at any time. What will happen to the mixture eventually?
2. A tank with a capacity of 500 gal originally contains 200 gal of water with 100 lb of salt in solution. Water containing 1 lb of salt per gallon is entering at a rate of 3 gal/min, and the mixture is allowed to flow out of the tank at a rate of 2 gal/min. Find the amount of salt in the tank at any time prior to the instant when the solution begins to overflow. Find the concentration (in pounds per gallon) of salt in the tank when it is on the point of overflowing. Compare this concentration with the theoretical limiting concentration if the tank had infinite capacity.
3. Suppose that a thermometer showing 22° inside the building is replaced outside where the temperature is 10° . After two minutes thermometer shows 16° . Find the function which shows the temperature of the thermometer at any time.
4. A cup of frappuccino is put under an AC which blows cold air with temperature changing as the function $\sin(t)$. If the temperature of frappuccino was 10° at the beginning, approximate the temperature of the beverage after 1 minute if $k = 1$.
5. A radioactive material is known to decay at a rate proportional to the amount present. If the half life of the material is 2000 years, find the amount of the material at any time t .
6. A ball with mass 0.15 kg is thrown upward with initial velocity 20 m/s from the roof of a building 30 m high.
 - a) Neglect the air resistance and find the velocity and position of the ball at time t . Plot the graphs of velocity and position versus time.
 - b) Assume that there is a force due to air resistance of magnitude $|v|/2$ directed opposite to the velocity, where the velocity v is measured in m/s. Find the velocity and position of the ball at time t . Find the maximum height above the ground that the ball reaches. What is the limiting velocity?
7. Suppose the population of whales in world's oceans increases by 20% per year in the absence of external factors. If there are originally 1000 whales and scientists are allowed to capture 150 whales per year, derive an IVP that gives the number of whales present in any year and solve it.
8. A certain insect population increases at a rate proportional to the square of the population. It is known that initially there are 1000 insects and after 10 days there are 5000 insects. Write an IVP and solve it.