MATH 505: Differentiable Manifolds
HOMEWORK 2
Due Date: April 5th, Friday

Problem 1: Let M be a smooth manifold and let $g : M \longrightarrow \mathbb{R}^+$ be a continuous fucntion. Show that there is a **smooth** function $f : M \longrightarrow \mathbb{R}^+$ such that $f(x) < g(x)$ for all $x \in M$.
(Hint: Use partition of unity on an open cover of M by coordinate balls.)

Problem 2: Lee, 3-4.

Problem 3: Lee, 3-5.

Problem 5:(i) Lee, 4-5(a).
(ii) Is every smooth quotient map a submersion?

Problem 5: Lee, 4-6.

Problem 6: Consider $F : \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ defined as $F(x, y, z) = (x^2-y^2, xy, xz, yz)$. Let $\phi = F_{|_{S^2}}$ where $S^2 \subset \mathbb{R}^3$ is the unit sphere. Observe that $\phi(p) = \phi(-p)$, so it gives a mapping $\tilde{\phi} : \mathbb{R}P^2 \longrightarrow \mathbb{R}^4$ where $\tilde{\phi}([p]) = \phi(p)$, $[p]$ = equivalence class of p.
(i) Show that $\tilde{\phi}$ is an injective immersion.
(ii) Is $\tilde{\phi}$ an embedding? Explain.