

**MATH 505: Differentiable Manifolds
HOMEWORK 1**

Due Date: March 13th, Wednesday

Problem 1: (i) Find countably many non-equal smooth structures on \mathbb{R} .
(ii) Lee, 1-6

Problem 2: Lee, 1-7.

Problem 3: Consider the subset N of \mathbb{R}^2 ,

$$N = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\} \cup \{(0, y) : 1 < y < 2\}$$

Find a smooth atlas of N with only one chart. Is the topology of N inherited from \mathbb{R}^2 ?

Problem 4: Consider a map $F : \mathbb{R}P^1 \rightarrow \mathbb{R}^2$ defined as

$$x = \frac{2t_1t_2}{(t_1)^2 + (t_2)^2}, \quad y = \frac{(t_1)^2 - (t_2)^2}{(t_1)^2 + (t_2)^2}$$

where $[t_1 : t_2]$ are the homogeneous coordinates on $\mathbb{R}P^1$.

(i) Show that F is well-defined and smooth.

(ii) Prove that the image of F is the unit circle S^1 and F gives a diffeomorphism between $\mathbb{R}P^1$ and S^1 .

(A similar map can be defined between $\mathbb{C}P^1$ and S^2 .)

Problem 5: Lee, 2-3 (c). What can you say about the preimage of a point under F ?

Problem 6: Lee, 2-5.