MATH 358: Partial Differential Equations
Problem Sheet 1

Problem 1. Find the partial differential equations (p.d.e.) which admit the followings as the general solution.

a) $u(x,y) = xy + f(xy)$

b) $u(x,y) = x^k f(y/x)$

Problem 2. Find partial differential equations which has the following family of surfaces as solutions.

a) $ax^2 + by^2 + u^2 = 1$

b) $u = (x^2 + a)(y^2 + b)$

Problem 3. Consider the p.d.e. $\dfrac{\partial u}{\partial x} + 2x\dfrac{\partial u}{\partial y} = y$

a) Use change of variables $s = s(x,y), t = t(x,y)$ to reduce the p.d.e. to an ordinary differential equation with respect to one of the variables.

b) Solve the o.d.e found in a) to find the general solution of the given p.d.e.

c) Find the solution to the Cauchy problem of p.d.e. with $u(x,0) = x^2, 0 < x < \infty$

Problem 4. In each of the following problems fins the solution surface which contains the given curve.

a) $\dfrac{\partial u}{\partial y} - xu\dfrac{\partial u}{\partial x} = 0, \quad u(x,0) = x.$

b) $u(\dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y}) = y - x, \quad u(x,0) = x.$

c) $x\dfrac{\partial u}{\partial x} - u\dfrac{\partial u}{\partial y} = x, \quad x + 2y = u, y = 2u.$

d) $x(y^2 + u)\dfrac{\partial u}{\partial x} - y(x^2 + u)\dfrac{\partial u}{\partial y} = (x^2 - y^2)u, \quad x + z = 0, y = 1.$

e) $(y + u)\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = x - y,\ y > 0 \quad u = x^2 + 1$ on $y = 1.$

Problem 5. Find the surfaces passing through the line $y = x, z = 1$ and orthogonal to the family of surfaces $x^2 + y^2 + z^2 = cx$.

Problem 6. a) Find all possible surfaces orthogonal to the planes $x + 2y + cz = 1$, where c is an arbitrary real constant.
(b) Find the surface orthogonal to the planes $x+2y+cz = 1$ passing through the curve $\Gamma : x = s, y = s, z = s^2$.

Problem 7. Determine whether the following Cauchy problems have solutions or not. If the solution exists, is it unique?

$$(y^2 - u)\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$$

a) $\Gamma_1 : \quad x = 1, y = s, z = s^2/2$
b) $\Gamma_2 : \quad x = s^2/2 - 1, y = s, z = s$