EXAMPLE 5.7

$ORIGIN := 1$

$T := 313$ $\quad$ $P := 0.1888$

$T_c := 514$ $\quad$ $P_c := 63$ $\quad$ $\omega := 0.644$

$T_r := \frac{T}{T_c} = 0.609$ $\quad$ $P_r := \frac{P}{P_c} = 2.997 \times 10^{-3}$

$$\alpha := \left[1 + \left(0.37464 + 1.54226 \cdot \omega - 0.26992 \cdot \omega^2\right) \cdot \left(1 - \sqrt{T_r}\right)\right]^2 = 1.628$$

$A := 0.45724 \cdot \left(\frac{P_r}{{T_r}^2}\right) \cdot \alpha = 6.015 \times 10^{-3}$ $\quad$ $B := 0.0778 \cdot \frac{P_r}{T_r} = 3.829 \times 10^{-4}$

$p := -1 + B$ $\quad$ $q := A - 3 \cdot B^2 - 2 \cdot B$ $\quad$ $r := -A \cdot B + B^2 + B^3$

$$M := Z^3 + p \cdot Z^2 + q \cdot Z + r \;\Big|\; \begin{matrix} \text{solve} \\ \text{assume}, Z = \text{real} \end{matrix} \rightarrow \begin{pmatrix} 0.0048274248899616859356 \\ 0.00044924479065136621964 \\ 0.99434045284284148405 \end{pmatrix}$$

$ZL := \min(M) = 4.492 \times 10^{-4}$ $\quad$ $ZV := \max(M) = 0.994$

$$\phi L := \exp\left[ZL - 1 - \ln(ZL - B) - \frac{A}{B \cdot \sqrt{8}} \cdot \ln\left[\frac{ZL + \left(1 + \sqrt{2}\right) \cdot B}{ZL + \left(1 - \sqrt{2}\right) \cdot B}\right]\right] = 0.994$$

$$\phi V := \exp\left[ZV - 1 - \ln(ZV - B) - \frac{A}{B \cdot \sqrt{8}} \cdot \ln\left[\frac{ZV + \left(1 + \sqrt{2}\right) \cdot B}{ZV + \left(1 - \sqrt{2}\right) \cdot B}\right]\right] = 0.994$$

ALTERNATIVE APPROACH

$T_c := 514$ $\quad$ $P_c := 63$ $\quad$ $\omega := 0.644$

$$\mathrm{root}(p, q, r) := \left| \begin{array}{l} v \leftarrow \begin{pmatrix} r \\ q \\ p \\ 1 \end{pmatrix} \\ x \leftarrow \mathrm{polyroots}(v) \\ \text{for } \; i \in 1..3 \\ \quad x_i \leftarrow 0 \;\; \text{if} \;\; \mathrm{Im}(x_i) \neq 0 \\ x1 \leftarrow \max(x) \\ y \leftarrow \min(x) \\ x2 \leftarrow \left| \begin{array}{l} \max(x) \;\; \text{if} \;\; y = 0 \\ y \;\; \text{otherwise} \end{array} \right. \\ \begin{pmatrix} x1 \\ x2 \end{pmatrix} \end{array} \right.$$

$$\phi(T, P) := \left| \begin{array}{l} T_r \leftarrow \dfrac{T}{T_c} \\ P_r \leftarrow \dfrac{P}{P_c} \\ \alpha \leftarrow \left[1 + \left(0.37464 + 1.54226\omega - 0.26992\omega^2\right)\cdot\left(1 - \sqrt{T_r}\right)\right]^2 \\ A \leftarrow 0.45724\cdot\dfrac{P_r\cdot\alpha}{T_r^{\,2}} \\ B \leftarrow 0.07780\cdot\dfrac{P_r}{T_r} \\ p \leftarrow -1 + B \\ q \leftarrow A - 2B - 3B^2 \\ r \leftarrow -A\cdot B + B^2 + B^3 \\ Z_V \leftarrow \mathrm{root}(p, q, r)_1 \\ Z_L \leftarrow \mathrm{root}(p, q, r)_2 \\ \Theta_V \leftarrow \dfrac{A}{\sqrt{8}\cdot B}\cdot\ln\left[\dfrac{Z_V + \left(1 + \sqrt{2}\right)\cdot B}{Z_V + \left(1 - \sqrt{2}\right)\cdot B}\right] \\ \Theta_L \leftarrow \dfrac{A}{\sqrt{8}\cdot B}\cdot\ln\left[\dfrac{Z_L + \left(1 + \sqrt{2}\right)\cdot B}{Z_L + \left(1 - \sqrt{2}\right)\cdot B}\right] \\ \phi_V \leftarrow \exp\left(Z_V - 1 - \ln\left(Z_V - B\right) - \Theta_V\right) \\ \phi_L \leftarrow \exp\left(Z_L - 1 - \ln\left(Z_L - B\right) - \Theta_L\right) \\ \begin{pmatrix} \phi_V \\ \phi_L \end{pmatrix} \end{array} \right.$$

Initial guess

$P := 0.1$

Given

$\phi(313, P)_1 = \phi(313, P)_2$

$\text{Find}(P) = 0.1888$