Taylor & Francis
Taylor & Francis Group

ANALYSIS OF THE ELECTROFILTRATION MECHANISM BASED ON MULTIPHASE FILTRATION THEORY

AYTEN GENÇ

Department of Environmental Engineering,
Zonguldak Karaelmas University,
Zonguldak, Turkey

İSMAIL TOSUN

Department of Chemical Engineering,
Middle East Technical University,
Ankara, Turkey

A mathematical model based on multiphase filtration theory is developed to describe the electrofiltration process. The model takes both electrophoretic and electroosmotic effects into account. The electrophoretic migration velocity of solid particles is predicted from the model using the filtrate volume time data obtained from electrofiltration experiments.

Keywords: Electrofiltration; Cake filtration; Multiphase filtration theory

INTRODUCTION

Electrofiltration is a technique that has been developed to enhance the performance of standard pressure filtration. The application of an electric field to a slurry retards the formation of cake at the surface of the filter medium and thereby increases the filtrate flow rate. Two electrokinetic phenomena are important in electrofiltration, namely, electrophoresis, in

which solid particles in liquid suspension migrate towards an oppositely charged electrode on the application of an electric field, and electro-osmosis, which enhances liquid flow through the cake. Although electroosmosis and electrophoresis were investigated many years ago (Reus, 1943), the mechanism of electrofiltration is still not well understood, even though some attempts have been made to develop mathematical models (Moulik et al., 1967; Yukawa et al., 1976; Iritani et al., 1992).

The first attempt at a mathematical model of electrofiltration was proposed by Moulik et al. (1967). They suggested that the electrofiltra-tion process is a combination of electrophoresis and filtration, and proposed a mathematical model based on the rate equation of standard filtration but with a correction to take into account the effect of electrophoresis. They performed electrofiltration experiments on two different concentrations of clay suspension (300 and 450 mg/L) and for different applied voltages. Their results showed that while for zero applied voltage (standard pressure filtration) the t/V versus V relation-ship was linear, for intermediate voltages the relationship was nonlinear. For higher voltages the relationship again became linear, and the voltage at which this transition occurred was defined as the critical voltage where the rate of electrophoretic migration of particles is equal to the linear rate of liquid flow through the filter medium and, therefore, there will be no cake formation at or above the critical voltage. According to this defi-nition of the critical voltage, Moulik and coworkers wrote the cake resistance occurring in electrofiltration as a factor $(1 - E/E_{cr})$ of the cake resistance occurring in filtration. They also proposed that electroosmosis increases the liquid rate through the filter medium and included this effect in the filter medium resistance. When these modified forms of the cake and medium resistances were substituted into the reciprocal filtrate rate equation, their results showed that the filtrate volume versus volume data obtained from the electrofiltration of clay suspension is linear.

Other electrofiltration models were derived by Yukawa et al. (1976) and Iritani et al. (1992). Yukawa and coworkers also represented the cake resistance occurring in electrofiltration in the same way as Moulik et al. (1967). The difference between these two models is in the way in which the effect of electroosmosis is accounted for. Yukawa and coworkers included the electroosmotic effect in the pressure term and subdivided the pressure into hydraulic and electroosmotic pressures. They showed experimentally that the electroosmotic pressure generated in the cake layer and filter medium is proportional to the strength of the electrical field. In addition, they concluded that the effect of electroosmosis is less than the effect of electrophoresis in cake filtration. Their electrofiltration model, which is similar to Ruth's filtration model, uses two parameters to describe the effects of electroosmosis and electrophoresis separately. Iritani et al. (1992) represented the cake growth rate in the electrofiltra-

tion by subtracting the mass flux of solids at the cake surface due to electrophoretic transport from the mass flux of solids at the cake surface due to convective transport. They evaluated the mass flux of the solids at the cake surface due to electrophoretic transport by multiplying the amount of solid in the suspension with the electrophoretic migration velocity of solid particles. Their model also has two parameters that show the effects of electroosmosis and electrophoresis separately. The values of these parameters can be evaluated experimentally and their experimental results show that in electrofiltration t/V versus V curves are nonlinear. When the models derived by Yukawa et al. (1976) and Iritani et al. (1992) are compared, it can be seen that the parameters that show the electro-osmotic effect in these two models are mathematically identical when the cake resistance is written in terms of the weight of wet cake.

The aforementioned electrofiltration models are all based on empirical filtration theory. This theory, however, fails to predict the actual mechanism of filtration since it yields unrealistic cake and filter medium resistances. The multiphase filtration theory of Willis and Tosun (1980) and Tosun (1986) explains the filtration mechanism in terms of the cake- filter medium interface permeability. According to this theory, the intercept of the reciprocal rate data is not related to the septum resistance but, rather, it is simply the initial reciprocal rate through the clean filter medium. The filter medium permeability appears in the slope of the reciprocal rate versus filtrate volume.

In the present study, a mathematical electrofiltration model is developed from multiphase filtration theory. The development of this model is described in detail and the calculation procedure used to evaluate the electrophoretic migration velocity of solid particles from the filtrate volume versus time data is presented.

EQUATIONS OF ELECTROFILTRATION

A schematic diagram of an electrofiltration unit with the slurry entering at the top and the filtrate leaving at the bottom is shown in Figure 1. For electrofiltration, the governing equations of continuity and motion, together with jump conditions, are given in Table I.

The jump conditions (Equations C, D, and G in Table I) are generally known as the Rankine-Hugoniot conditions. The superscripts $+$ and $-$ represent the limits as the cake-slurry interface is approached from the slurry side $(z - L > 0)$, and from the cake side, $(z - L < 0)$, respectively.

A constitutive relation for the drag force, F_{d_z}, occurring in pressure filtration can be postulated in the form

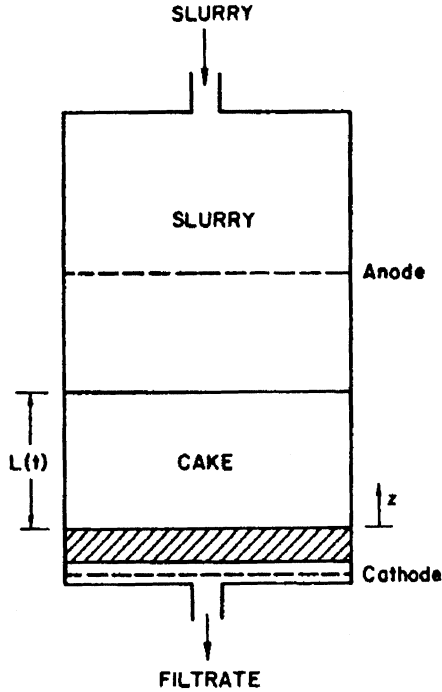$$F_{d_z} = \lambda(v_f - v_s) \tag{1}$$

Figure 1. Schematic diagram of an electrofiltration unit.

where λ is the interface drag coefficient. The drag force occurring in electrofiltration will be assumed to be described in the same manner but with a different interface drag coefficient. Substituting the interface drag force coefficient into Equation (E) in Table I gives

$$\varepsilon v_f - \varepsilon v_s = \frac{\varepsilon^2}{\lambda_e} \frac{\partial P_e^*}{\partial z} \tag{2}$$

where λ_e and P_e^* represent the interface drag force coefficient and the piezometric pressure in electrofiltration, respectively. The piezometric pressure is defined as

$$P_e^* = P_e + \rho g z \tag{3}$$

where P_e is the summation of the hydraulic and electroosmotic pressures. Evaluating Equation (2) at the cake-filter medium interface ($z = 0$) gives

$$\frac{dt}{dV} = \frac{L}{A} \left[\frac{(\lambda_e/\varepsilon)_{z=0}}{J_o \Delta P_e} \right] \tag{4}$$

Table I Equations of Change and Jump Conditions for Electrofiltration (Genç, 1996)

Equations of Continuity
Liquid Phase

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial(\varepsilon v_f)}{\partial z} \tag{A}$$

Solid Phase

$$-\frac{\partial \varepsilon}{\partial t} = \frac{\partial[(1-\varepsilon)v_s]}{\partial z} \tag{B}$$

Jump Conditions at the Cake-Slurry Interface

$$\frac{dL}{dt} = \frac{\varepsilon^+ v_f^+ - \varepsilon^- v_f^-}{\varepsilon^- - \varepsilon^+} \tag{C}$$

$$\frac{dL}{dt} = \frac{(1-\varepsilon^+)v_s^+ - (1-\varepsilon^-)v_s^-}{\varepsilon^+ - \varepsilon^-} \tag{D}$$

Equations of motion
Liquid Phase

$$\varepsilon\left(\frac{\partial P_e}{\partial z} - \rho g\right) - F_{d_z} = 0 \tag{E}$$

Solid Phase

$$(1-\varepsilon)\left(\frac{\partial P_e}{\partial z} - \rho g\right) - F_{d_z} + \frac{\partial P_s}{\partial z} + (1-\varepsilon)(\rho_s - \rho)g = 0 \tag{F}$$

Jump Conditions at the Cake-Slurry Interface

$$P_s^- = P_s^+ \tag{G}$$

where J_o is the dimensionless pressure gradient defined by

$$J_o = \frac{L}{\Delta P_e^*}\left(\frac{\partial P_e^*}{\partial z}\right)_{z=0} \tag{5}$$

To obtain the liquid velocity in the cake region, the continuity equation for the liquid, Equation (A) in Table I, is integrated over the volume of filter cake:

$$\int_0^{L(t)} \frac{\partial \varepsilon}{\partial t}\,dz = \int_0^{L(t)} \frac{\partial(\varepsilon v_f)}{\partial z}\,dz \tag{6}$$

The integral on the left-hand side of Equation (6) is evaluated using Leibnitz's rule:

$$\frac{d}{dt} \int_0^{L(t)} \varepsilon \, dz - \varepsilon^- \frac{dL}{dt} = \varepsilon^- v_f^- - (\varepsilon v_f)_{z=0} \tag{7}$$

With the boundary condition at the cake-filter medium interface, i.e.,

$$(\varepsilon v_f)_{z=0} = \frac{1}{A} \frac{dV}{dt} \tag{8}$$

and the average porosity, $\langle \varepsilon \rangle$, defined as

$$\langle \varepsilon \rangle = \frac{1}{L} \int_0^L \varepsilon \, dz \tag{9}$$

Equation (7) reduces to

$$\varepsilon^- v_f^- = \frac{1}{A} \frac{dV}{dt} - (\varepsilon^- - \langle \varepsilon \rangle) \frac{dL}{dt} \tag{10}$$

Substituting Equation (10) into the jump conditions at the cake-slurry interface, Equation (C) of Table I, gives the velocity of liquid in the slurry region as

$$v_f^+ = \frac{1}{\varepsilon^+ A} \frac{dV}{dt} - \left(\frac{\varepsilon^+ - \langle \varepsilon \rangle}{\varepsilon^+} \right) \frac{dL}{dt} \tag{11}$$

In a similar manner the velocity of solid particles in the slurry region can be determined as:

$$v_s^+ = \left(\frac{\varepsilon^+ - \langle \varepsilon \rangle}{1 - \varepsilon^+} \right) \frac{dL}{dt} \tag{12}$$

In the slurry region the difference between the interstitial liquid and solid velocities is equal to the electrophoretic migration velocity of the solid particles, ϑ_e, i.e.,

$$v_f^+ - v_s^+ = \vartheta_e \tag{13}$$

Substitution of Equations (11) and (12) into Equation (13) yields

$$\frac{1}{A}\frac{dV}{dt} = \frac{1}{\phi}\frac{dL}{dt} + \varepsilon^+\vartheta_e \tag{14}$$

where

$$\phi = \frac{1 - \varepsilon^+}{\varepsilon^+ - \langle\varepsilon\rangle} \tag{15}$$

Integrating Equation (14) with respect to time and assuming constant electrophoretic migration velocity gives

$$L = \phi\left(\frac{V}{A} - \vartheta_e\varepsilon^+t\right) \tag{16}$$

Substitution of Equation (16) into Equation (4) gives

$$\frac{dt}{dV} = \beta\left(\frac{V}{A} - \vartheta_e\varepsilon^+t\right) \tag{17}$$

where

$$\beta = \frac{\phi(\lambda_e/\varepsilon^2)_{z=0}}{J_o\Delta P_e^*A^2} \tag{18}$$

The solution of Equation (17) subject to initial condition, $V|_{t=0} = 0$, is

$$t = \frac{1}{\vartheta_e\varepsilon^+A}\left(V - \frac{1}{\beta\vartheta_e\varepsilon^+A}\right) + \frac{1}{(\vartheta_e\varepsilon^+A)^2\beta}\exp(-\beta\vartheta_e\varepsilon^+AV) \tag{19}$$

Polynomial expansion of the exponential term by Taylor series up to the fourth degree gives

$$\frac{t}{V} = \frac{\beta}{2}V - \frac{\beta^2\eta}{6}V^2 + \frac{\beta^3\eta^2}{24}V^3 \tag{20}$$

where

$$\eta = \vartheta_e\varepsilon^+A \tag{21}$$

Equation (20) indicates that t/V versus V curves in electrofiltration ($\eta \neq 0$) are nonlinear. The parameters β and η can be found by fitting the t/V versus V curves to data obtained from electrofiltration experiments.

EXPERIMENTAL APPARATUS AND PROCEDURE

The experimental setup is shown schematically in Figure 2. The filter chamber has a circular cross-sectional area of $37.39\,cm^2$. Slurry at a
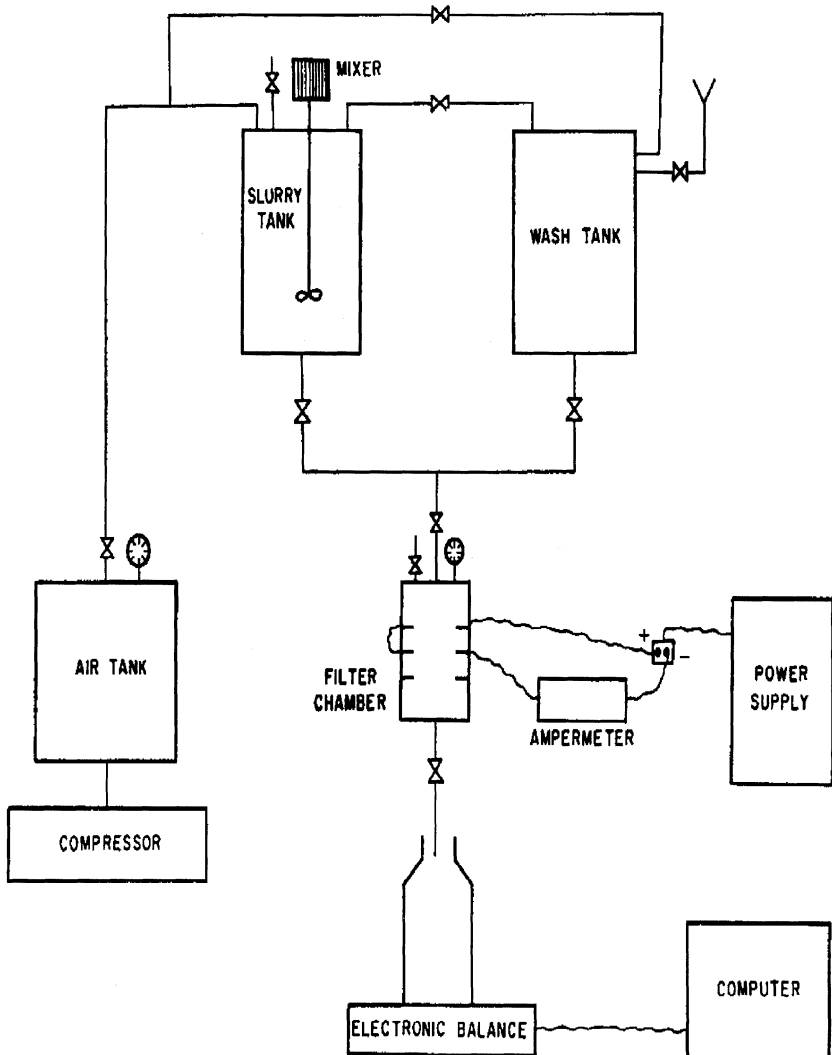

Figure 2. Experimental setup.

constant pressure of 1.7 atm was forced through the filter chamber having a cellulose acetate membrane with 0.2 μm pore size for a fixed period of time (30 min). The slurry used for the experiments was 1% by weight of anatase (TiO_2) particles (average particle size 0.3 μm) in water. The electric field was applied through the electrodes from a d.c. power supply (400 V and 10 A). The variation of filtrate volume with time was measured by an electronic balance at 10-second time intervals. These experiments are reported in greater detail in Genç (1996).

RESULTS AND DISCUSSIONS

The results obtained from the filtration of 1% of anatase (TiO_2) particles without applying a voltage difference to the electrodes are shown in the form of t/V versus V in Figure 3.

The t/V versus V curves obtained from experiments at two different electric field strengths (E = 50, 67 V/cm) are shown in Figure 4. Cubic polynomials were fitted to the experimental data and resulted in the following equations:

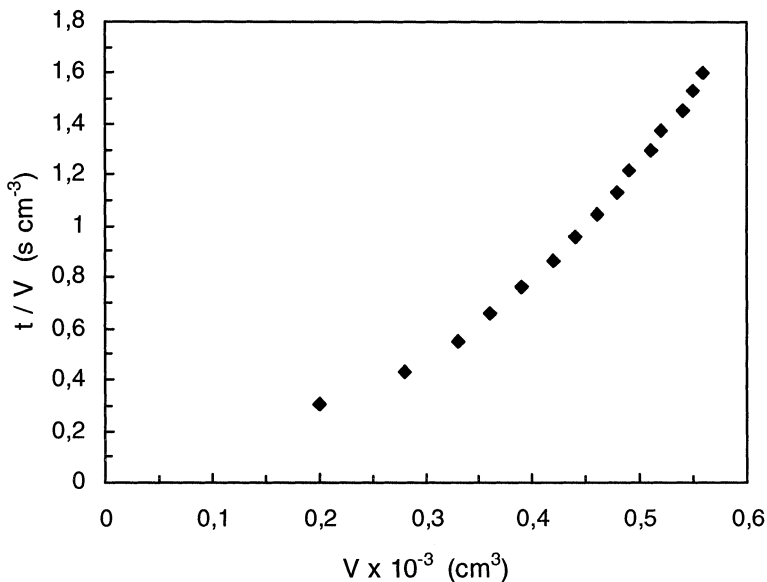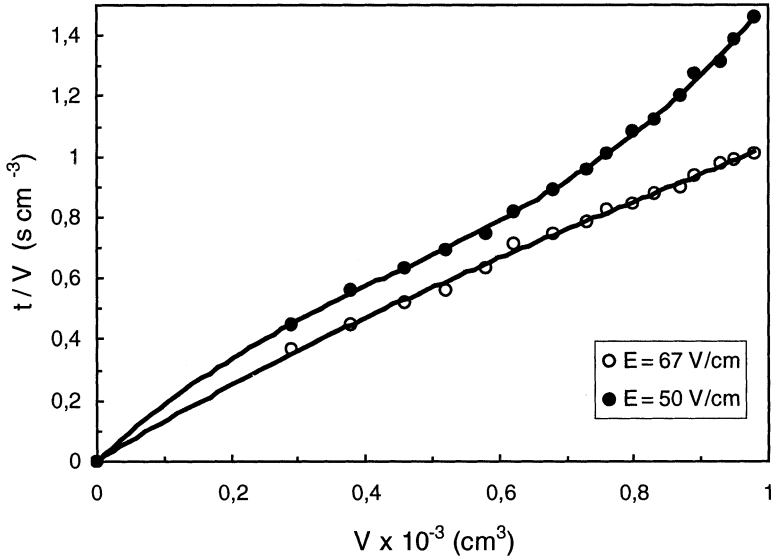**Figure 3.** t/V versus V values for 1% anatase suspension (pH = 6, P = 1.7 atm, E = 0 V/cm).

Figure 4. t/V versus V values for 1% anatase suspension (pH = 6, P = 1.7 atm.)

$$\frac{t}{V} = 1.88 \times 10^{-3}\,V - 1.97 \times 10^{-6}\,V^2 + 1.548 \times 10^{-9}\,V^3 \quad \text{for } E = 50\,V/cm$$

$$\frac{t}{V} = 1.55 \times 10^{-3}\,V - 1.21 \times 10^{-6}V^2 + 7.094 \times 10^{-10}V^3 \quad \text{for } E = 67\,V/cm$$

$$(22)$$

The values of β and η were calculated by comparing coefficients of Equation (20) with those of Equation (22) and the results are given in Table II. Note that η values are more sensitive than β values to the electric field strength.

Once the numerical value of η is known, the electrophoretic migration velocity of solid particles can be determined from Equation (21). For very dilute suspensions it can be assumed that ε^+ is equal to the porosity in the feed tank, and for a solid concentration of 1%, $\varepsilon^+ = 0.997$.

TABLE II Numerical Values of β and η

Electric Field Strength (V/cm)	β	η
50	0.00376	0.836
67	0.00310	0.756

Table III Calculated and Measured values of ϑ_e

Electric Field Strength (V/cm)	$\vartheta_e (cm/s)$	
	Experimental	Calculated
50	0.0202	0.0240
67	0.0224	0.0317

The calculated electrophoretic migration velocities of anatase particles are compared with those obtained from the zeta potential measurements (Zeta Meter System 3+) in Table III. Note that the mathematical model gives fairly good estimates of the electrophoretic migration velocity of anatase particles.

CONCLUSIONS

Experimental results show that for electrofiltration the t/V versus V curves are nonlinear, and this is consistent with the mathematical model developed in this paper. The electrophoretic migration velocity of solid particles can be evaluated from t/V versus V curves using the electrofiltration model developed. The calculated electrophoretic migration velocities of anatase particles are shown to be in good agreement with those obtained from zeta potential measurements.

NOMENCLATURE

A	cross-sectional area, m^2
E	electric field strength, V/cm
E_{cr}	critical electric field strength, V/cm
F_{d_z}	z-component of the drag force for unit volume, N/m^3
g	gravitational acceleration, m/s^2
J_o	dimensionless pressure gradient, defined by Equation (5)
L	cake length, m
P_e	summation of hydraulic and electroosmotic pressure in electrofiltration, Pa
P_e^*	piezometric pressure in electrofiltration defined by Equation (3), Pa
P_s	solids compressive pressure, Pa
t	time, s
V	volume of filtrate, m^3
v	velocity, m/s
z	axial coordinate, m

Greek letters

β	function defined by Equation (18), s/m^6
ε	volume fraction
$\langle \varepsilon \rangle$	average porosity defined by Equation (9)
λ	interface drag coefficient in filtration, $Pa.s/m^2$

λ_e interface drag coefficient in electrofiltration, Pa.s/m^2
η function defined by Equation (21), m^3/s
ρ liquid density, kg/m^3
ϑ_e electrophoretic migration velocity of solid, m/s
ϕ function defined by Equation (15)

Subscripts
f liquid
s solid

Superscripts
+ above the cake-slurry interface
− below the cake-slurry interface

REFERENCES

Genç, A. (1996). Effect of electrode configuration in electrofiltration, Ph.D. diss., Middle East Technical University, Ankara, Turkey.

Iritani, E., Ohashi, K., and Murase, T. (1992). *J. Chem. Eng. Jpn.*, **25**, 383.

Moulik, S. P., Cooper, F. C., and Bier, M. (1967). *J. Colloid Interface Sci.*, **24**, 427.

Reus, H. J. (1943). *Principles and Applications of Electrochemistry*, 4th ed., Vol. 1, Chapman and Hall, London, pp. 145−165.

Tosun, İ. (1986). *R & D Innovations in Filtration*, J. Hermia, ed., Belgian Filtration Society, Louvain-La-Neuve, Belgium, pp. 255−268.

Willis, M. S. and Tosun, İ. (1980). *Chem. Eng. Sci.*, **35**, 2427.

Yukawa, H., Kobayashi, K., Tsukui, Y., Yamano, S., and Iwata, M. (1976). *J. Chem. Eng. Jpn.*, **9**, 396.