DUFFIE and BECKMAN CHAPTER 3

Speed of light in vacuum: $C_o := 3 \cdot 10^{8} \frac{m}{s}$

Note that C= Co / n = λ v

For a medium with $n := 1.2$ $C := \frac{C_o}{n} = 2.5 \cdot 10^{8} \frac{m}{s}$

For solar energy applications, the range of wavelength is between UV and near-infrared from 0.29 μm to 25 μm

$\lambda\_min := 0.29\ \mu m$ $\lambda\_max := 25.0\ \mu m$

In the medium with n=1 index of refraction, that range corresponds to the frequency range of $n := 1$

$C := \frac{C_o}{n} = 3 \cdot 10^{8} \frac{m}{s}$

$\nu\_max := \frac{C}{\lambda\_min} = 1.0345 \cdot 10^{15}\ Hz$ $\nu\_min := \frac{C}{\lambda\_max} = 12000000000000\ Hz$

Visible light is in the wavelength range of

$\lambda\_vis\_min := 0.38\ \mu m$ Violet $\lambda\_vis\_max := 0.78\ \mu m$ Red

$\nu\_vis\_max := \frac{C}{\lambda\_vis\_min} = 7.895 \cdot 10^{14}\ Hz$ $\nu\_vis\_min := \frac{C}{\lambda\_vis\_max} = 3.846 \cdot 10^{14}\ Hz$

Planck's Constant $h = 6.626 \cdot 10^{-34}\ s\ J$

Photon energy range of visible light from E= h v

$E\_vis\_max := h \cdot \nu\_vis\_max = 5.231 \cdot 10^{-19}\ J$ $E\_vis\_min := h \cdot \nu\_vis\_min = 2.548 \cdot 10^{-19}\ J$

Planck's Law radiation constants $C_1 := 3.7405 \cdot 10^{8}\ W \frac{\mu m^4}{m^2}$ $C_2 := 14387.8\ \mu m\ K$

Wien's displacement constant $C_3 := 2897.8\ \mu m\ K$

Taking equivalent blackbody temperature of the sun as $T_{sun} := 5777\ K$

From Wien"s displacement $\lambda\_max\_sun := \frac{C_3}{T_{sun}} = 0.5016\ \mu m$

From Planck's Law blackbody emissive power of the sun at its maximum wavelength

$$E\_\lambda b\_max\_sun := \frac{C_1}{\lambda\_max\_sun^5 \cdot \left(\exp\left(\frac{C_2}{\lambda\_max\_sun \cdot T_{sun}} \right) - 1 \right)} = 8.276 \cdot 10^{7} \frac{W}{m^2\ \mu m}$$

See Figure 3.4.1

Stefan-Boltzmann Constant $\sigma := 5.6697 \cdot 10^{-8} \frac{W}{m^2 K^4}$

Total blackbody emissive power of the sun $E\_b\_sun := \sigma \cdot T_{sun}^4 = 6.315 \cdot 10^7 \frac{W}{m^2}$

Radiation fraction function for λT can be obtained for a blackbody at 5777 K

Let's calculate the fraction of the visible range:

From 0 to 0.38 μm (violet) $\lambda T1 := \lambda\_vis\_min \cdot T_{sun} = 2195 \ \mu m \ K$

$\gamma := \frac{C_2}{\lambda T1} = 6.554$

$$f\_0\_\lambda T1 := \frac{15}{\pi^4} \cdot \sum_{nn=1}^{10} \left(\frac{\exp(-nn \cdot \gamma)}{nn} \cdot \left(\gamma^3 + \frac{3 \cdot \gamma^2}{nn} + \frac{6 \cdot \gamma}{nn^2} + \frac{6}{nn^3} \right) \right) = 0.1000$$

From 0 to 0.78 μm (red) $\lambda T2 := \lambda\_vis\_max \cdot T_{sun} = 4506.06 \ \mu m \ K$

$\gamma := \frac{C_2}{\lambda T2} = 3.193$

$$f\_0\_\lambda T2 := \frac{15}{\pi^4} \cdot \sum_{nn=1}^{10} \left(\frac{\exp(-nn \cdot \gamma)}{nn} \cdot \left(\gamma^3 + \frac{3 \cdot \gamma^2}{nn} + \frac{6 \cdot \gamma}{nn^2} + \frac{6}{nn^3} \right) \right) = 0.5652$$

Therefore from 0.38 μm to 0.78 μm

$$f\_\lambda T1\_\lambda T2 := f\_0\_\lambda T2 - f\_0\_\lambda T1 = 0.4652$$

Blackbody emissive power in the visible range is $E\_b\_vis := f\_\lambda T1\_\lambda T2 \cdot E\_b\_sun = 2.938 \cdot 10^7 \frac{W}{m^2}$

From Eq. (3.9.2) Effective sky temperature can be calculated

Let's calculate the sky temperature for Ankara at 20:02 on 02.02.2020 using the weather data for Guvercinlik:

$Ta := 9 \ °C = 282.2 \ K$ $tt := 20$ $Tdp := 2$ Tdp is in Celcius but to make the units match taken as unitless

$$Ts := Ta \cdot \left(0.711 + 0.0056 \cdot Tdp + 0.000073 \cdot Tdp^2 + 0.013 \cdot \cos(15 \cdot tt) \right)^{\frac{1}{4}} = 260.1 \ K$$

$$DeltaT := Ta - Ts = 22.05 \ K$$