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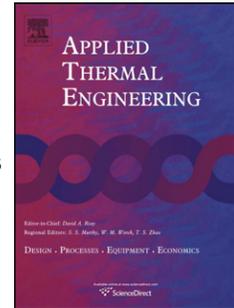
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**A CORRELATION FOR NATURAL CONVECTION HEAT TRANSFER
FROM INCLINED PLATE-FINNED HEAT SINKS**

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Abstract

Steady state natural convection heat transfer from inclined plate-finned heat sinks to air is numerically investigated by using an experimentally validated model. The heat sinks with parallel arrangement of uniform rectangular cross section plate fins are inclined from the vertical in both forward and backward directions in order to investigate the effect of inclination on convection. Our previously validated numerical model for vertically oriented heat sinks is directly used without changing any model parameters, but only by varying the direction of the gravitational acceleration to create the effect of inclination. The flow and temperature fields are resolved using a finite volume computational fluid dynamics code. Performing a large number of simulations for the heat sink base inclination angles of $\pm 4^\circ$, $\pm 10^\circ$, $\pm 20^\circ$, $\pm 30^\circ$, $\pm 45^\circ$, $\pm 60^\circ$, -65° , -70° , $\pm 75^\circ$, $\pm 80^\circ$, $\pm 85^\circ$, $\pm 90^\circ$ from the vertical, the dependence of the convective heat transfer rate to the inclination angle and Rayleigh number is investigated. Scale analyses are performed in order to generalize estimates for the convection heat transfer rates. A single correlation is suggested and shown to be valid for a very wide range of angles from -60° (upward) to $+80^\circ$ (downward) in a wide range of Rayleigh numbers from 0 to 2×10^8 .

Keywords: Natural convection; Plate fin array; Inclined heat sink; Electronics cooling; Correlation.

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Nomenclature

d	Base plate thickness, m	S	Fin spacing, m
g	Gravitational acceleration, ms^{-2}	S_{opt}	Optimum fin spacing, m
Gr	Grashof number	t	Fin thickness, m
\bar{h}	Average convective heat transfer coefficient, $\text{Wm}^{-2}\text{K}^{-1}$	T_a	Ambient temperature, $^{\circ}\text{C}$
H	Fin height, mm	T_f	Film temperature, $^{\circ}\text{C}$, $T_f=(T_w+T_a)/2$
k	Thermal conductivity of air, $\text{Wm}^{-1}\text{K}^{-1}$	T_w	Average heat sink temperature, $^{\circ}\text{C}$
L	Heat sink length, mm	ΔT	Base-to-ambient temperature difference, $^{\circ}\text{C}$
N	Number of fins	W	Heat sink width, mm
\overline{Nu}_L	Average Nusselt number based on L , $\overline{Nu}_L = \bar{h}L / k$	α	Thermal diffusivity, m^2s^{-1}
Pr	Prandtl number	β	Volumetric thermal expansion coefficient, K^{-1}
Q_c	Total convection heat transfer rate from fin array, W	θ	Angle of inclination with respect to vertical orientation, deg ($^{\circ}$)
Q_{in}	Power supplied to heater plate, W	ν	Kinematic viscosity, m^2s^{-1}
Ra	Rayleigh number based on L , $Ra=g\beta L^3(T_w-T_a)/(\alpha\nu)$		

1. Introduction

Heat sinks are often used in electronics cooling when high heat fluxes should be dissipated. In most of the electronics cooling applications, the final heat transfer medium is air, thus, the forced convection takes

precedence over the natural convection. For the purposes of conserving energy and increasing reliability, however, the natural convection may be the right choice; especially, if the generated heat can be spread over a larger surface using heat spreaders or heat pipes. Recently, possibilities of dissipating enough heat by natural convection and radiation were shown for the vertical orientation of plate-finned heat sinks in two separate applications [1][2]. In general, the vertical and upward facing horizontal orientations of a heat sink are preferred for maximizing the natural convection heat transfer rate [3][4]. As a result, these two orientations were investigated extensively. In contrast, there are only a few works investigating inclined orientations [5][6][7].

Inclined orientations are important for at least two reasons: one, vertical or horizontal surfaces may not be available due to design constraints; two, an originally vertical or horizontal heat sink may become inclined during its operation when the cooled electronic device is rotated, intentionally or otherwise. The possibility of intentional or unintentional rotation by itself is enough a reason to investigate inclined orientations because of the associated decline from the maximum heat transfer rate, which is only obtainable either for the vertical or the upward horizontal cases.

In the case of forced convection, the possible heat dissipation rate from a heat sink highly depends on the remaining components of the cooling system, such as the fan and the enclosure; therefore, obtaining general dimensionless heat transfer correlations is not possible. In contrast, in natural convection, it is possible to make generalizations and obtain application independent correlations. Consequently, in literature, there are many correlations for vertical and horizontal orientations, which, however, considerably vary from study to study.

Our contribution

Recently, in Tari and Mehrtash [7], we suggested a set of three correlations relating average Nusselt number based on fin spacing (S) to a modified Rayleigh number where a modified Grashof number was defined based on major fin dimensions, namely spacing (S), height (H) and length (L). In the present study, improving upon our former work, we unify these three correlations by proposing an alternative one. In our new correlation, we collect all of the geometric parameters on the left hand side and relate the heat transfer rate to the commonly used Rayleigh number based on length. Our new correlation agrees very well with all available experimental data in literature for the inclined case (Starner and McManus [5] and Mittelman *et al.* [6]). Additionally, in

order to better analyze the phenomenon for inclinations close to horizontal, we perform new simulations covering additional inclination angles.

Related literature

For rotated plates, Elenbaas [8] conducted the first comprehensive experimental work, where he proposed that g should be replaced by $g \cos\theta$ in the calculation of the average Nusselt number (Nu) because $g \cos\theta$ is the only component that causes the flow of air when the plates are rotated by an angle θ . Note, however, that this reasoning is inappropriate for the finned heat sinks due to corner geometries involved. Despite its importance, there are only a few works investigating the inclined orientations of the finned heat sinks. Starner and McManus [5] were probably the first ones to investigate the effect of base orientation in three different cases (vertical, 45° inclination and horizontal) on the rate of heat transfer from rectangular fin arrays. They found that the measured heat transfer coefficients for the vertical orientation were generally lower than those for parallel-plate channels investigated by Elenbaas [8]. The inclined orientation (45°) comes with a reduction in the heat transfer coefficient as a result of blocking. The horizontal orientation yielded more favorable results than the 45° inclination due to ample flow from above down to the array.

For downward facing inclined orientations, Mittelman *et al.* [6] performed a combined experimental, analytical and numerical study in order to investigate the natural convection for inclinations of 60°, 70°, 80° and 90° with respect to the vertical. They reported that the optimum fin spacing for horizontal fin arrays is applicable to the inclined ones; furthermore, this optimum spacing does not depend on the inclination angle. Additionally, their experiments indicate that the heat dissipation rate is not significantly affected by the changes in the fin height.

The only two existing experimental inclined plate finned heat sink studies, namely Starner and McManus [5] and Mittelman *et al.* [6], cover only upward 45° and downward 60°, 70° and 80°, respectively, whereas the present study fills the literature gap related to inclination angles, by numerical determination of heat transfer rates from plate-finned heat sinks within a wide range of inclination angles.

In contrast to the inclined case, the literature on the vertical and horizontal cases is too rich to be completely covered within the scope of the present paper; a sampling includes [9]-[15], among others. Comprehensive reviews, nevertheless, can be found in Dialameh *et al.* [16], Harahap *et al.* [17], Dogan and Sivrioglu [18].

2. Numerical simulations

The model assembly used in the present work is schematically depicted in Fig. 1. It includes an aerated concrete insulation, a heater plate, and a heat sink attached to the heater. The assembly in Fig. 1 is placed in an air filled cubical room of 3 m sides with walls that are maintained at uniform 20 °C. In the analysis (with ANSYS Fluent), steady state solutions are obtained by using the zero-equation-turbulence model with initial ambient air temperature of 20 °C. Air is taken as an ideal gas at atmospheric pressure. No slip boundary condition is used for all surfaces. There is no contact resistance between solid surfaces. The dimensions and the material properties for each of the components are presented in Table 1 for two considered heat sinks. A more detailed description of the numerical model, the model validation processes (for a flat plate and a vertical fin array), and the effect of different parameters on the rate of heat transfer rate in the vertical case were previously given in [7]. Thus, we briefly review the solution method in the next paragraph, and then focus on generalization of the data obtained for the inclined cases using a scale analysis.

A non-conformal mesh structure with a very fine grid around the cooling assembly and a coarse grid for the rest of the room is employed. Grid independence is achieved by examining three different grid densities with 1685832, 2834264 and 4077608 cells, and then selecting the medium density mesh, i.e., the one with 2834264 cells, as it yields results matching to those of the fine mesh. ANSYS Fluent solver is used for solving the continuity, momentum and thermal energy equations for air and the heat conduction equation within the solids. To handle the radiative heat transfer, the surface-to-surface radiation model is used.

Steady-state heat-transfer performance of vertical rectangular fins protruding from a base that is oriented in various angles of inclination has been assessed in [7]. The present study is an extended investigation covering inclination angles of $\pm 4^\circ$, $\pm 10^\circ$, $\pm 20^\circ$, $\pm 30^\circ$, $\pm 45^\circ$, $\pm 60^\circ$, -65° , -70° , $\pm 75^\circ$, $\pm 80^\circ$, $\pm 85^\circ$, $\pm 90^\circ$ from the vertical. Here, by following a similar procedure to the one in Bejan and Lee [19], we obtain an expression for predicting the maximum possible rate of convection heat transfer based on geometric parameters for the vertical orientation of plate-finned heat sinks. The obtained correlation for the vertical case is modified as suggested for an inclined

parallel plate array by Elenbaas [8], and then the validity range of the modified correlation is tested using the simulation data obtained at the considered inclination angles.

3. Vertical case results and discussion

Vertical case simulations in [7] were performed for two different fin lengths of 250 and 340 mm. Parameters that were kept constant are the fin thickness ($t=3$ mm), the base thickness ($d=5$ mm), and the base-plate width ($W=180$ mm). Fin height and fin spacing values were varied from 5 to 25 mm and 5.75 to 85.5 mm, respectively. In addition to these geometrical parameters, five heat input values ranging from 25 to 125 W were used for all the fin configurations. Optimum fin spacing values corresponding to the highest rates of convection heat transfer from different fin array configurations were calculated. The optimum number of fins spanning the width of 250 mm is determined to be $N=13$ corresponding to the fin spacing value of 11.75 mm. A scale analysis similar to the one in Bejan and Lee [19] has been performed on the numerical data in order to estimate the convection heat transfer rate from the vertical fin array (Q_c). When the derivation in [19] for obtaining a correlation form is applied to our heat sink geometry the form of the dimensionless convective heat transfer rate becomes

$$\frac{Q_c}{kH\Delta T(W/L)} = CRa^n \quad (1)$$

where Q_c is the convective heat transfer rate; ΔT is the base-to-ambient temperature difference; k is the thermal conductivity of air; H , L and W are the fin height, the heat sink length and the heat sink width, respectively. On the right hand side of equation (1), Ra is the Rayleigh number based on heat sink length. C and n are dimensionless constants yet to be determined from the data. Note that the geometry in [19] is similar to the one in Elenbaas [8], rather than plate finned heat sink geometry, but our form (which is different than the one in [19]) is particularly meaningful in the heat sink setting, as indicated by the following observation:

$$\frac{Q_c}{kH\Delta T(W/L)} = \frac{\bar{h}A\Delta T}{k\Delta T} \frac{L}{HW} = \frac{\bar{h}L}{k} \frac{A}{HW} = \overline{Nu}_L \left(\frac{L}{H} \right) \left(\frac{A}{LW} \right) \quad (2)$$

In the rightmost form, the first term \overline{Nu}_L is the average Nusselt number based on heat sink length, the second term L/H is the aspect ratio of the fin geometry, and the third term $A/(LW)$ is the finned area to base area ratio of the heat sink. We prefer the leftmost form because Q_c is explicit in the respective expression while it is implicit in the rightmost form. This makes Q_c extraction more practical.

In order to accommodate fin height dependence, the left hand side of (1) is scaled with a factor obtained from the analysis of the simulation data which is $(H/L)^{0.32}$. Figs. 2 and 3 show the variations of $\{Q_c/[kH\Delta T(W/L)]\}(H/L)^{0.32}$ with respect to the Rayleigh number based on length (Ra) defined as $Ra=g\beta L^3(T_w-T_a)/(\alpha\nu)$ in which all fluid properties are evaluated at the film temperature, T_f . Curve fit results are also provided in order to demonstrate the trend of the data points for each fin height. In Fig. 2, all of the data obtained for the fin spacing values between 8.8-14.7 mm have been clustered into three distinct groups corresponding to each fin height. The squared correlation coefficients (R^2) of the equations shown for $H=5, 15,$ and 25 mm are respectively 0.976, 0.9914, and 0.9602.

Observing that the data for $H=5$ mm shows contrasting behaviour with the data from the other two fin heights and considering the fact that the ratio of $H/L=5/250=0.02$ is unusually small for a heat sink (*i.e.* resembles a flat plate), the data for vertical rectangular fin arrays of the present study was generalized and presented only for $H=15$ and 25 mm in Fig. 3. The resulting correlation equation is

$$\frac{Q_c}{kH\Delta T(W/L)}\left(\frac{H}{L}\right)^{0.32} = 1.24Ra^{0.385} \quad (3)$$

The squared correlation coefficient (R^2) of 0.9511 indicates that equation (3) is a very good form to fit to our data.

4. Inclined case results and discussion

The validated numerical model for investigating natural convection from a plate-fin heat sink in vertical orientation is directly used for inclined case investigations; only the direction of the gravitational acceleration vector is varied in order to simulate both backward and forward inclined orientations in the range of 0-90°.

Simulations are conducted at 22 different angles with respect to the vertical: $\pm 4^\circ$, $\pm 10^\circ$, $\pm 20^\circ$, $\pm 30^\circ$, $\pm 45^\circ$, $\pm 60^\circ$, -65° , -70° , $\pm 75^\circ$, $\pm 80^\circ$, $\pm 85^\circ$, $\pm 90^\circ$. The minus sign indicates the upward facing direction, and the plus sign the downward facing one. Three different power input values of 25, 75, 125 W are supplied to the heater plate. The heat sink length is set to 250 mm, and the fin spacing to the optimum value ($S_{opt} = 11.75$ mm) which has been obtained in the vertical case. Out of five geometric parameters, only the fin height is varied, from 5 to 25 mm as in the vertical case, while all the others are kept constant.

In Fig. 4, variations of the convective heat transfer rate from the fin array (Q_c) as a function of inclination angle are plotted for all the three fin heights for $Q_m = 75$ W, in both downward and upward directions. We make the following observations:

- The highest Q_c values are achieved at the vertical orientation, confirming the observation of Starner and McManus [5].
- In the $0-30^\circ$ range for both directions of inclination, Q_c changes very slowly, staying nearly constant.
- Up to 60° of inclination, the downward case Q_c values are slightly higher than the upward case ones. This can be attributed to the thinning effect on the boundary layers within the fin channels due to downward facing orientation of the heat sink.
- For all of the three fin heights in the downward case, Q_c changes monotonically until the inclination gets very close to $+90^\circ$, with inflection points observed around 80° . This supports the observation by Mittelman *et al.* [6] that there is a substantial enhancement in the rate of heat transfer trend beyond a certain angle of inclination starting from the downward horizontal.
- The behavior of the Q_c as a function of the inclination changes at around $\theta = 60^\circ$, after which the minimum is observed. The angle at which the minimum of Q_c is observed depends on the fin height (H). In fact for $H = 5$ mm, the minimum is observed at the horizontal. The increase in the heat transfer rate after achieving the minimum is steeper for the larger fin heights due to larger surface areas which entail higher convective heat transfer rates.

- In the upward case, as H increases, the difference between the Q_c values for the vertical ($\theta=0^\circ$) and the upward horizontal ($\theta=-90^\circ$) cases gets smaller.

In Figs. 5 and 6, respectively for the downward and upward inclinations, the streamlines around the heat sink with $H=25$ mm and $Q_{in}=125$ W are shown on a plane parallel to the base, *i.e.*, y - z cross section. Each sub-figure depicts the streamlines at a different inclinations angle, seven in the downward case and six in the upward case. The speed scale is kept the same in all of the sub-figures. In the upward case (Fig. 5), flow separation starts to appear on the heat sink (the smaller rectangle attached to the bigger insulation) at around $\theta=-60^\circ$. As the angle of inclination increases beyond -60° , the separation location move from the top edge towards the center. On the other hand, in the downward case (Fig. 6), the separation starts to appear on the heat sink at around $\theta=80^\circ$, and then move from the bottom edge towards the center of the heat sink. Whenever the separation is on the heat sink, we observe two flows from (for upward inclinations) or to (for downward inclinations) the top and bottom sides of the heat sink. These flows are symmetric at both $\theta=-90^\circ$ and $\theta=90^\circ$.

In Figs. 7 and 8, respectively in the downward and upward directions, the speed contours of the flow at 85° of inclination are shown on a plane perpendicular to the base, *i.e.*, x - y cross section, at 2 mm above the base surface. Each of the three subfigures depicts the speed contours for each of the three fin heights in order to inspect the effect of fin height on the flow inside the heat sink. The speed scale is the same for all of the figures. The red horizontal lines show the locations of flow separation in each channel. The obvious effect of using a larger fin height is an increase in steady state convective heat dissipation from the heat sink as a result of the increase in the cooling surface area. In addition, larger fin heights cause the flow separation to first appear on the heat sink at a slightly smaller inclination angle, apparent from the separation locations at 85° inclination appearing farther from the bottom edge (Fig. 7). This observation corroborates with a recent assertion by Mittelman *et al.* [6], which implies that a larger fin height simultaneously reduces sensitivity, increases the driving force, and increases friction thus enhancing the movement of the flow separation line toward the center of the heat sink. According to Fig. 7 (downward inclination), flow separations take place in all channels approximately at the same location. But, this does not happen when the heat sink is tilted in the other direction (Fig. 8). In Fig. 8 (upward inclination), the location of these points are almost the same for $H=25$ mm, but for

smaller fin heights, $H=15$ mm and especially in $H=5$ mm, the flow separation locations differ from channel to channel. From the middle channels towards the sides, the separation points move towards the upper edge, showing that the separation starts earlier in the middle, and indicating that flow instabilities occur in the side channels. Obviously, a heat sink orientation that has a gravity component in the upward direction is more susceptible to instability relative to an orientation that has a downward gravity component. For a heat sink with a smaller fin height, this instability increases due to reductions in friction and the driving force (buoyancy). Moreover, for the smaller fin heights in the upward case, there are flows slightly above the fins from the sides of the heat sink towards the center that force the flow inside the channels to rotate and detach (forming longitudinal vortices).

In addition to inclined orientations, we have also tested our numerical model at the both horizontals ($\theta=90^\circ$ and $\theta=-90^\circ$). We have examined various fin configurations differing in fin spacing. The optimum fin spacing values for minimizing the horizontal heat sink surface average temperature are presented in Table 2. While the optimum fin spacing in the vertical case is around 12 mm, in the upward facing horizontal case, it is around 13 mm, and in the downward facing horizontal case, it is around 9 mm. It can be deduced from these values that the optimum fin spacing values stay within the interval of 8.8-14.7 mm. Therefore, equation (3), the vertical case correlation obtained for this fin spacing interval, is expected to be valid after a modification by multiplying Ra on the right hand side with $\cos\theta$.

$$\frac{Q_c}{kHDT(W/L)}\left(\frac{H}{L}\right)^{0.32} = 1.24(Ra \cos\theta)^{0.385} \quad (4)$$

Of course, provided that equation (4) is only being considered up to certain inclination angles which are to be determined. Since, undoubtedly, equation (4) would not account for the flow separation location, a phenomenon that does not occur in the vertical case, the vertical correlations modified this way are not expected to be valid after flow separation starts to occur on the heat sink. Hence, equation (4) should only be considered up to -60° and $+80^\circ$ for the upward and the downward cases respectively.

In Figs. 9 and 10, respectively in the downward and upward directions, the variation of

$\{Q_c/[kHAT(W/L)]\}(H/L)^{0.32}$ is plotted against $Ra \cos\theta$ for the present inclined case simulation data. In each of the

two figures, equation (4) is shown with a dashed line. The simulation data shows excellent agreement with equation (4) within the inclination angle interval of $-60^\circ \leq \theta \leq +80^\circ$.

We further verify our suggested correlation using experimental results by Mittelman *et al.* [6] for downward inclinations and Starner and McManus [5] for an upward inclination of 45° ; these constitute all of the available inclined case data in the literature. The Mittelman *et al.* [6] data is in very good agreement with equation (4). See Fig. 11 where the correlation curve is plotted together with the data from both Mittelman *et al.* and our simulations in downward 60° - 80° range. An equivalent verification for Starner and McManus [5] data is depicted in Fig. 12. The experimental data agree very well with our correlation. Note that both Mittelman *et al.* [6] and Starner and McManus [5] data are collected for quite different geometric parameters and conditions from both each other's and ours. Thus, their agreement with equation (4) is an indication of the generality of our suggested correlation.

5. Conclusion

Improving upon our former work, Tari and Mehrtash [7], for estimating convection heat transfer rates from plate-finned heat sinks inclined from the vertical, we suggest a new correlation covering a wide range of inclination angles, $-60^\circ \leq \theta \leq +80^\circ$, reducing the set of three correlations in [7] to a single one unifying different Rayleigh number ranges. Our new correlation is verified using all of the available inclined case experimental data in the literature.

Analyzing our own simulation data, which is an extension of our former data in [7] –including a larger number of inclination angles covering upward inclinations in detail as well as fin spacing optimizations for the upward and downward horizontal orientations–, we made the following observations:

- The optimum fin spacing for all inclinations varies between 9 mm for the downward facing horizontal to 13 mm for the upward horizontal, the optimum for the vertical being in the middle. As a result, using the optimum fin spacing for the vertical orientation is advisable if there is a possibility of inclination of the heat sink.

- The suggested correlation, equation (4), is valid in $-60^\circ \leq \theta \leq +80^\circ$ range in which flow structure stays similar to the one in the vertical case.
- Flow separation occurs on the heat sink for the angles in $-90^\circ \leq \theta < -60^\circ$ and $+80^\circ < \theta \leq +90^\circ$. At these angles close to the horizontal orientations, flow separation and fin height play the most significant roles in the natural convection heat transfer capability of an inclined plate-finned heat sink.
- When the data for the heat sink with $H=5$ mm is examined, some flow instabilities are observed for the upward inclinations that are very close to the horizontal. Since these instabilities may cause unexpectedly low heat dissipation rates, one must be cautious while selecting the fin height for the upward inclined heat sinks; *i.e.*, fin height should be large enough to avoid instabilities.

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FIGURE CAPTIONS

Fig. 1. Schematic view of each component of the model assembly separately.

Fig. 2. Data obtained for $H=5, 15,$ and 25 mm (S is in $8.8-14.7$ mm interval) along with the proposed correlations for each one.

Fig. 3. Correlation obtained for $H=15$ and 25 mm (these are very practical fin heights for electronics cooling applications).

Fig. 4. Convection heat transfer rates obtained in upward and downward inclinations for $H=5, 15,$ and 25 mm.

Fig. 5. Streamlines in and around the heated fin array ($H=25$ mm) for various downward inclination angles:

(a) $\theta=0^\circ$; (b) $\theta=45^\circ$; (c) $\theta=60^\circ$; (d) $\theta=75^\circ$; (e) $\theta=80^\circ$; (f) $\theta=85^\circ$; (g) $\theta=90^\circ$.

Fig. 6. Streamlines in and around the heated heat sink ($H=25$ mm) for various upward inclination angles:

(a) $\theta=0^\circ$; (b) $\theta=-45^\circ$; (c) $\theta=-60^\circ$; (d) $\theta=-75^\circ$; (e) $\theta=-85^\circ$; (f) $\theta=-90^\circ$.

Fig. 7. Speed contours at $\theta=85^\circ$ for various fin heights: (a) $H=25$ mm; (b) $H=15$ mm; (c) $H=5$ mm at 2 mm above the base plate surface.

Fig. 8. Speed contours at $\theta=-85^\circ$ for various fin heights: (a) $H=25$ mm; (b) $H=15$ mm; (c) $H=5$ mm at 2 mm above the base plate surface.

Fig. 9. Downward inclined case results for $H=15$ and 25 mm plotted together with equation (4).

Fig. 10. Upward inclined case results for $H=15$ and 25 mm plotted together with equation (4).

Fig. 11. Simulation results and Ref. [6] results in downward $60^\circ-80^\circ$ inclination angle range plotted together with equation (4).

Fig. 12. Simulation results and Ref. [5] results at -45° plotted together with equation (4).

TABLES

Table 1 Properties of the components

Component	Dimensions		Material Properties				
	Setup 1	Setup 2	Material Type	Specific Heat (J/kg°C)	Conductivity (W/mK)	Emissivity	Roughness (mm)
Heat sink	180×250×5	180×340×5	Aluminum	900	130	0.2	0.02
Heater base plate	180×250×5	180×340×5	Aluminum	900	130	0.2	0.02
Concrete block	340×450×100	340×450×100	Aerated concrete	1000	0.15	0.9	2

Table 2 Estimates for optimum fin spacing minimizing the average temperature at base horizontally oriented

Q_{in} (W)	Optimum Fin Spacing at Horizontal Orientation, S_{opt} (mm)					
	Facing Down			Facing Up		
	$H=25$ mm	$H=15$ mm	$H=5$ mm	$H=25$ mm	$H=15$ mm	$H=5$ mm
25	9.3	9.1	8.8	13.9	13.6	12.4
75	9.7	9.4	9.2	12.8	13.5	13.8
125	9.9	9.6	9.6	12.3	13.5	13.9

FIGURES

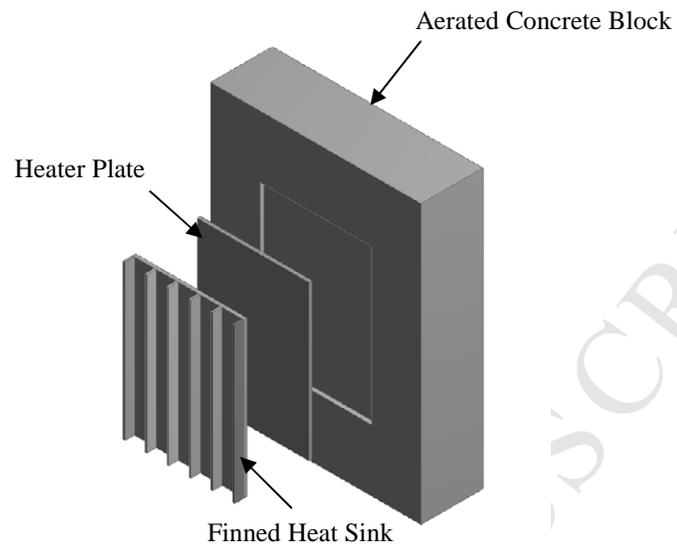


Fig. 1. Schematic view of each component of the model assembly separately.

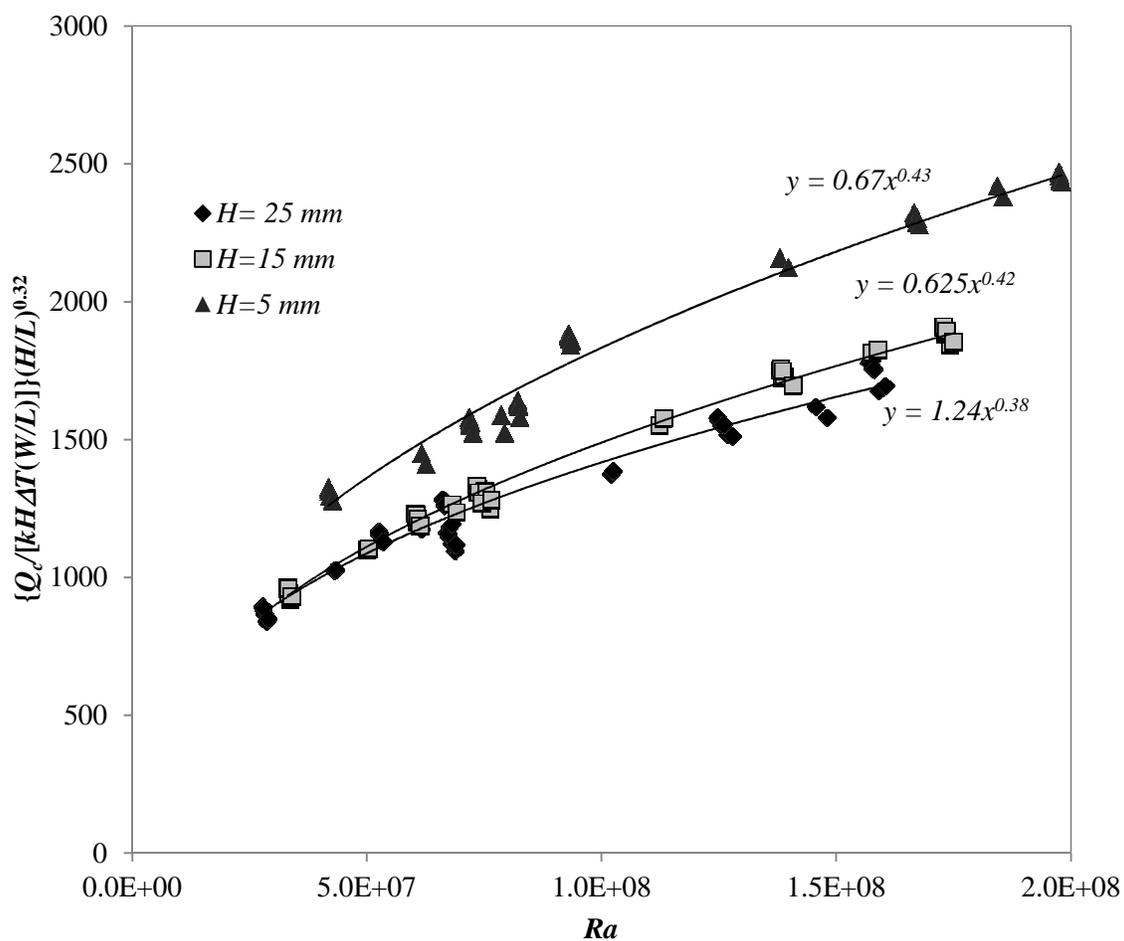


Fig. 2. Data obtained for $H=5, 15,$ and 25 mm (S is in $8.8-14.7$ mm interval) along with the proposed correlations for each one.

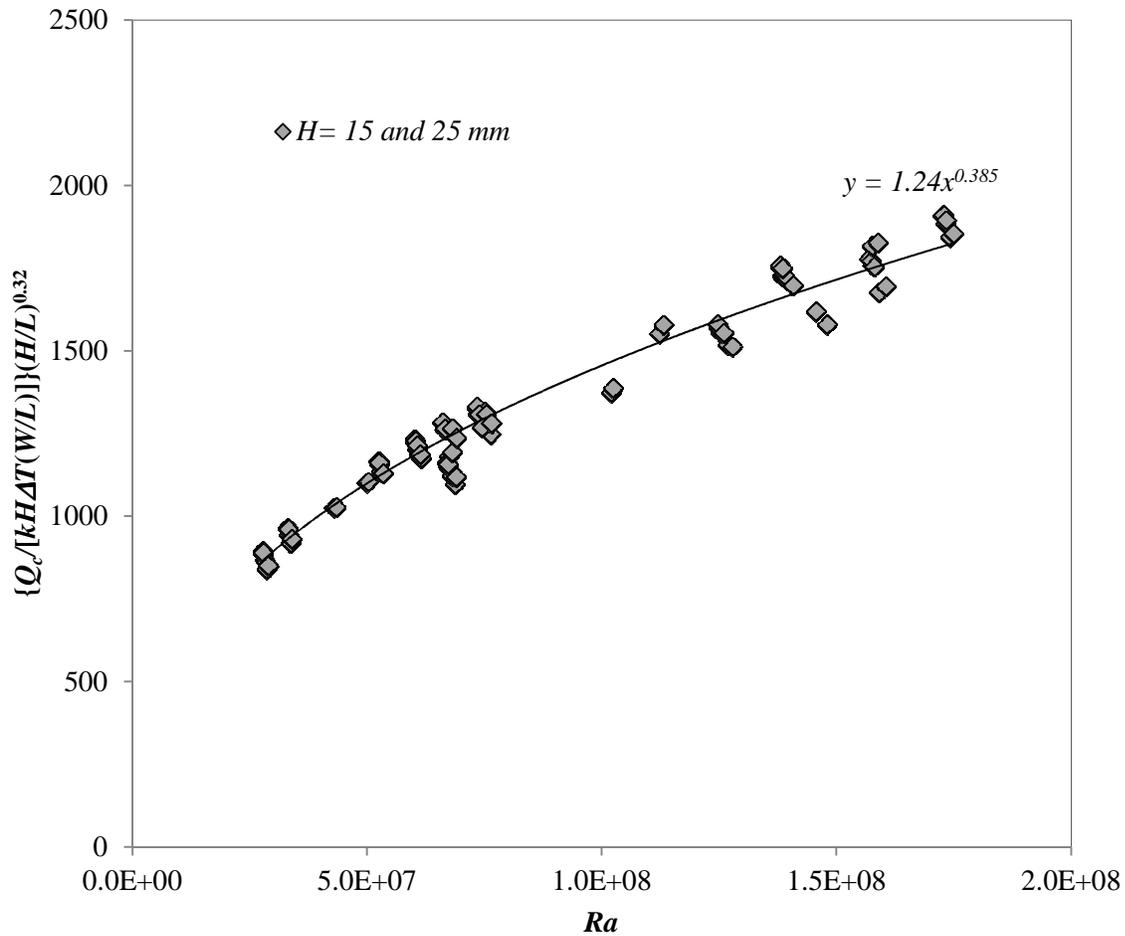


Fig. 3. Correlation obtained for $H=15$ and 25 mm (these are very practical fin heights for electronics cooling applications).

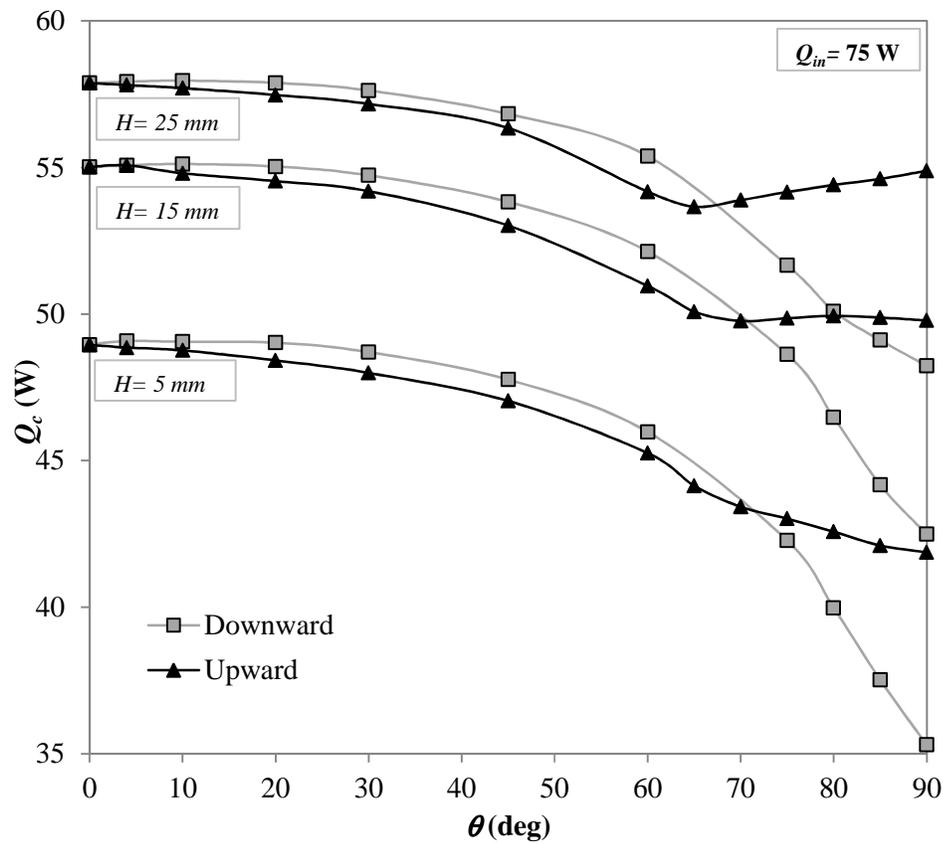


Fig. 4. Convection heat transfer rates obtained in upward and downward inclinations for $H=5$, 15, and 25 mm.

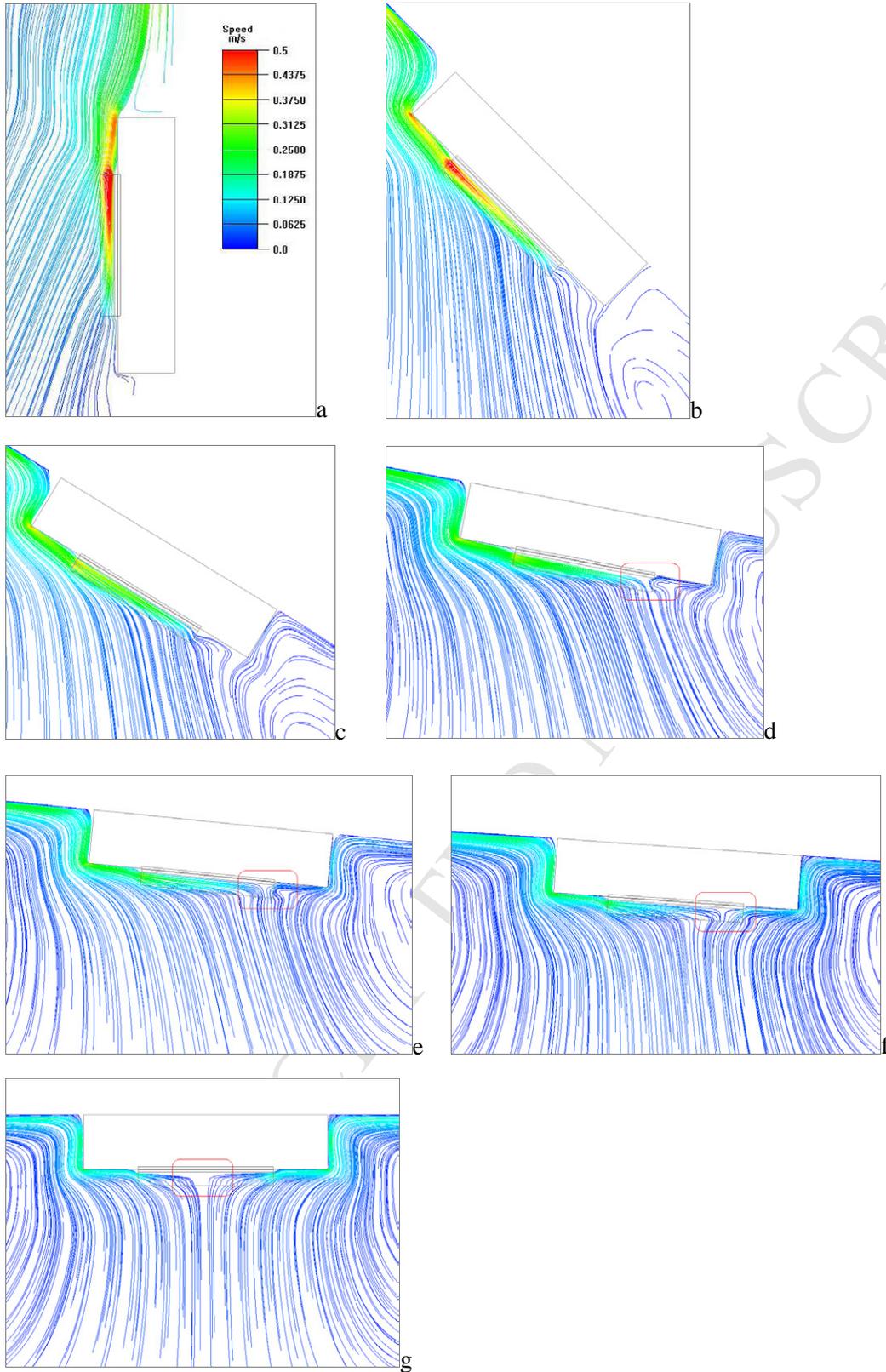


Fig. 5. Streamlines in and around the heated fin array ($H=25$ mm) for various downward inclination angles:

(a) $\theta=0^\circ$; (b) $\theta=45^\circ$; (c) $\theta=60^\circ$; (d) $\theta=75^\circ$; (e) $\theta=80^\circ$; (f) $\theta=85^\circ$; (g) $\theta=90^\circ$.

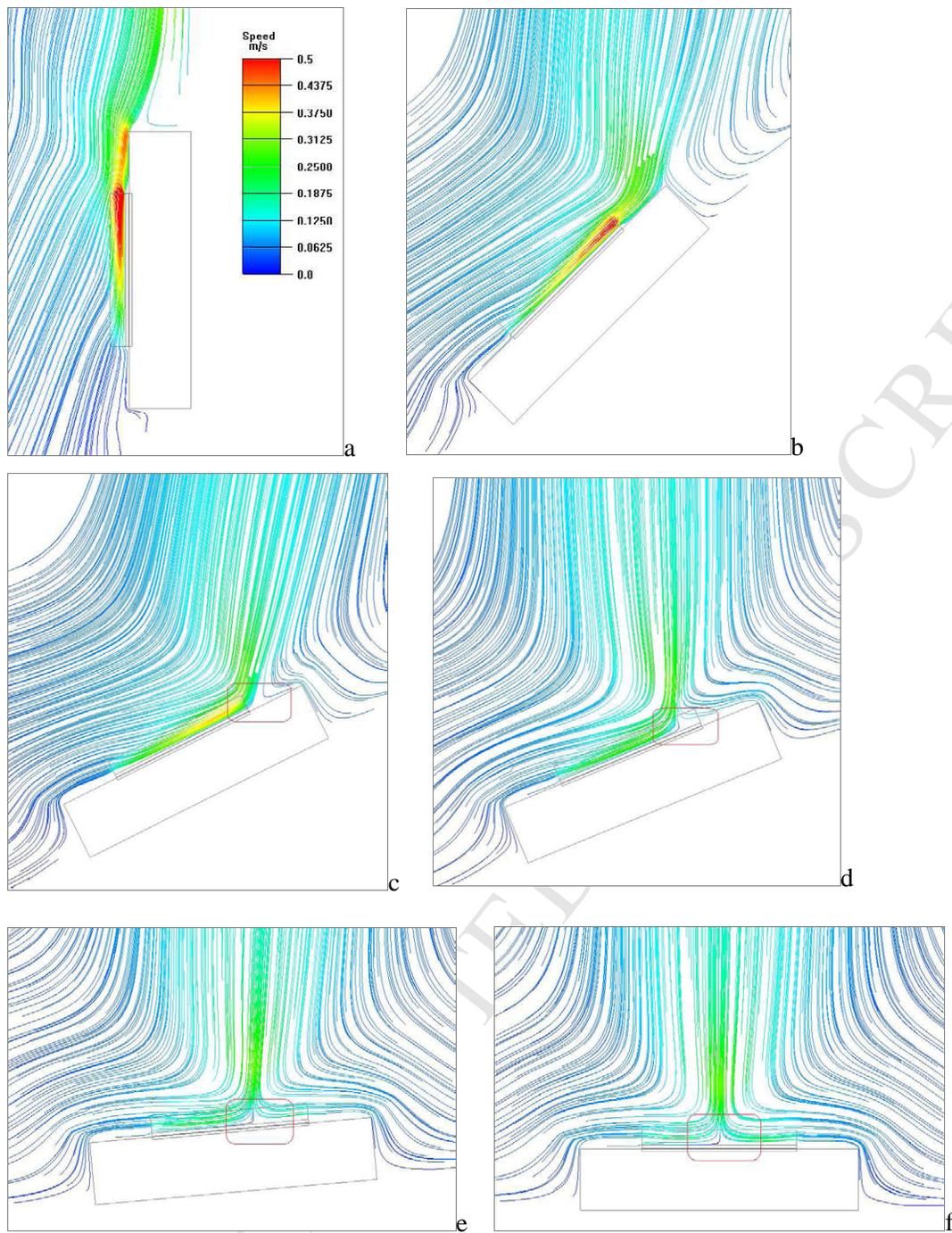


Fig. 6. Streamlines in and around the heated heat sink ($H=25$ mm) for various upward inclination angles:

(a) $\theta=0^\circ$; (b) $\theta=-45^\circ$; (c) $\theta=-60^\circ$; (d) $\theta=-75^\circ$; (e) $\theta=-85^\circ$; (f) $\theta=-90^\circ$.

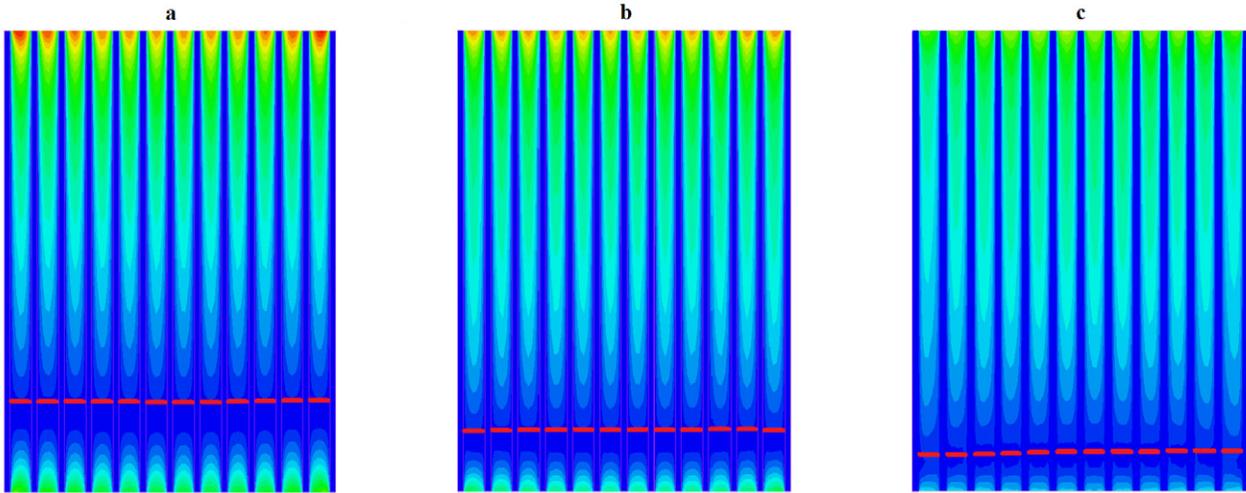


Fig. 7. Speed contours at $\theta=85^\circ$ for various fin heights: (a) $H=25$ mm; (b) $H=15$ mm; (c) $H=5$ mm at 2 mm above the base plate surface.

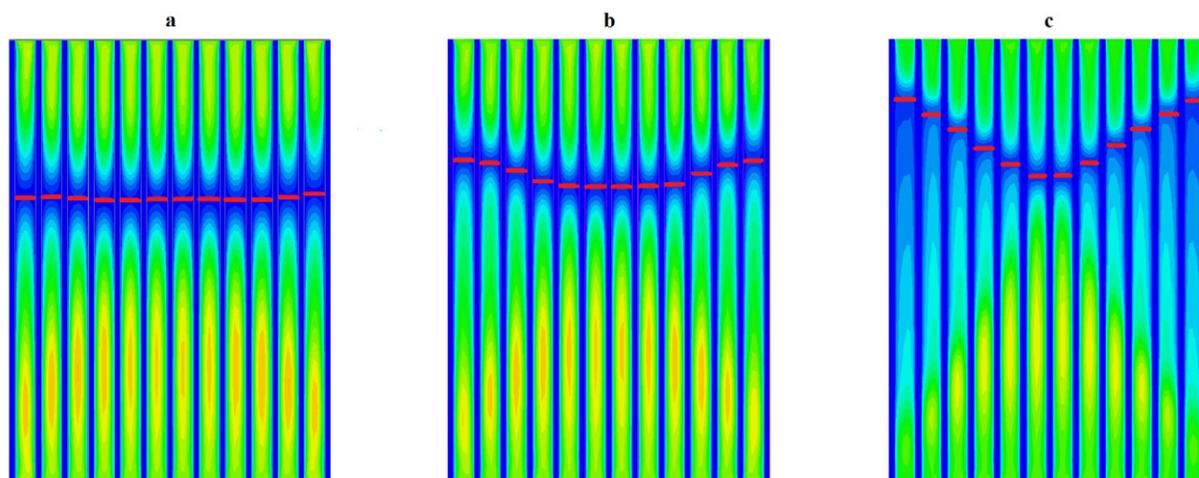


Fig. 8. Speed contours at $\theta=-85^\circ$ for various fin heights: (a) $H=25$ mm; (b) $H=15$ mm; (c) $H=5$ mm at 2 mm above the base plate surface.

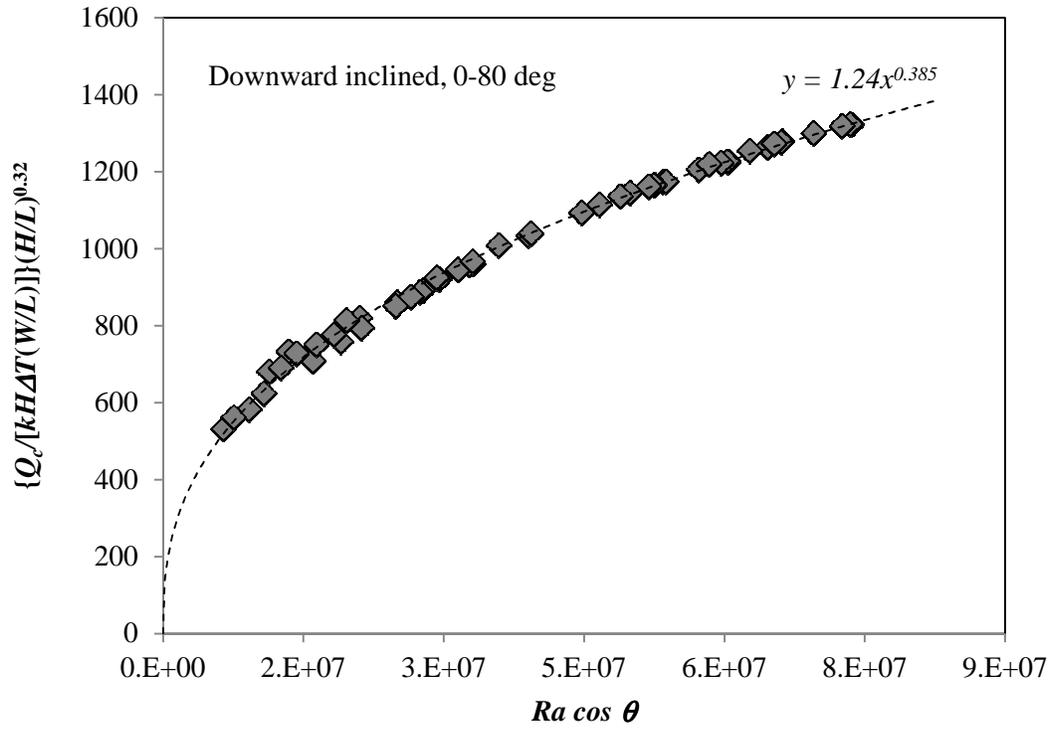


Fig. 9. Downward inclined case results for $H=15$ and 25 mm plotted together with equation (4).

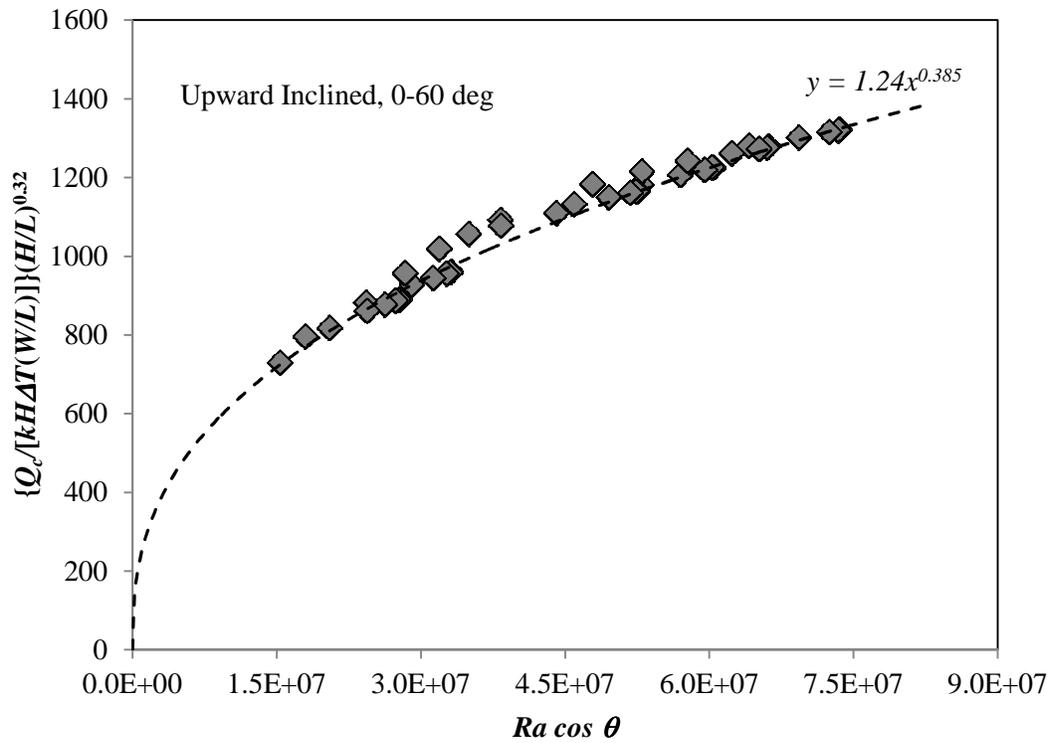


Fig. 10. Upward inclined case results for $H=15$ and 25 mm plotted together with equation (4).

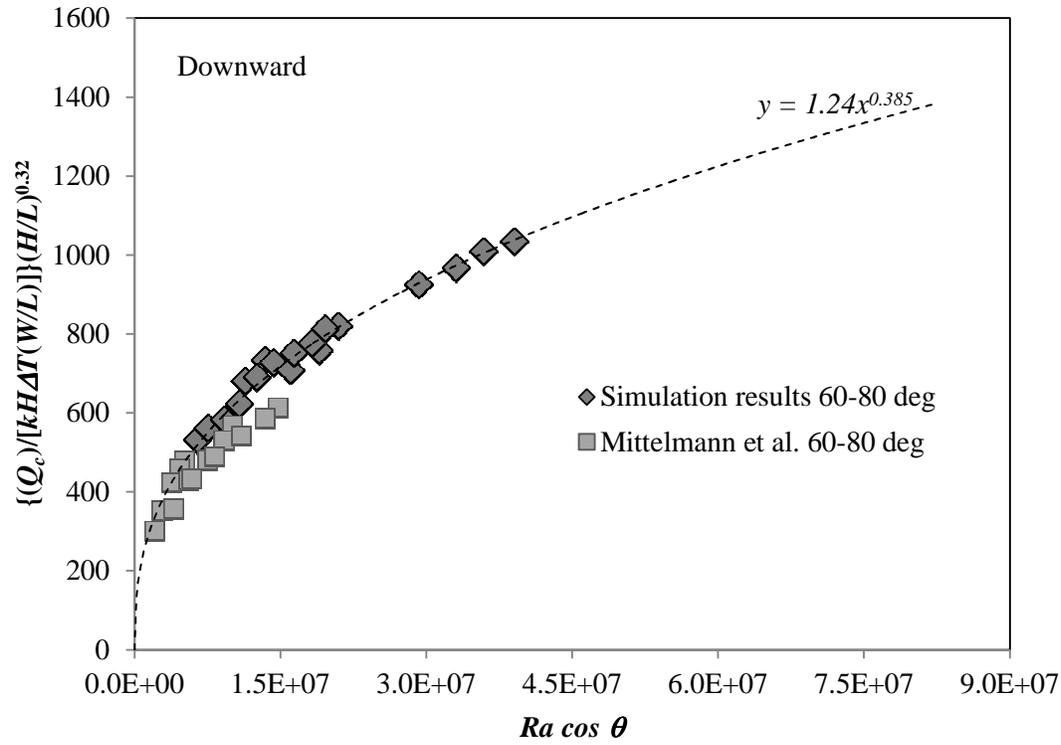


Fig. 11. Simulation results and Ref. [6] results in downward 60°-80° inclination angle range plotted together with equation (4).

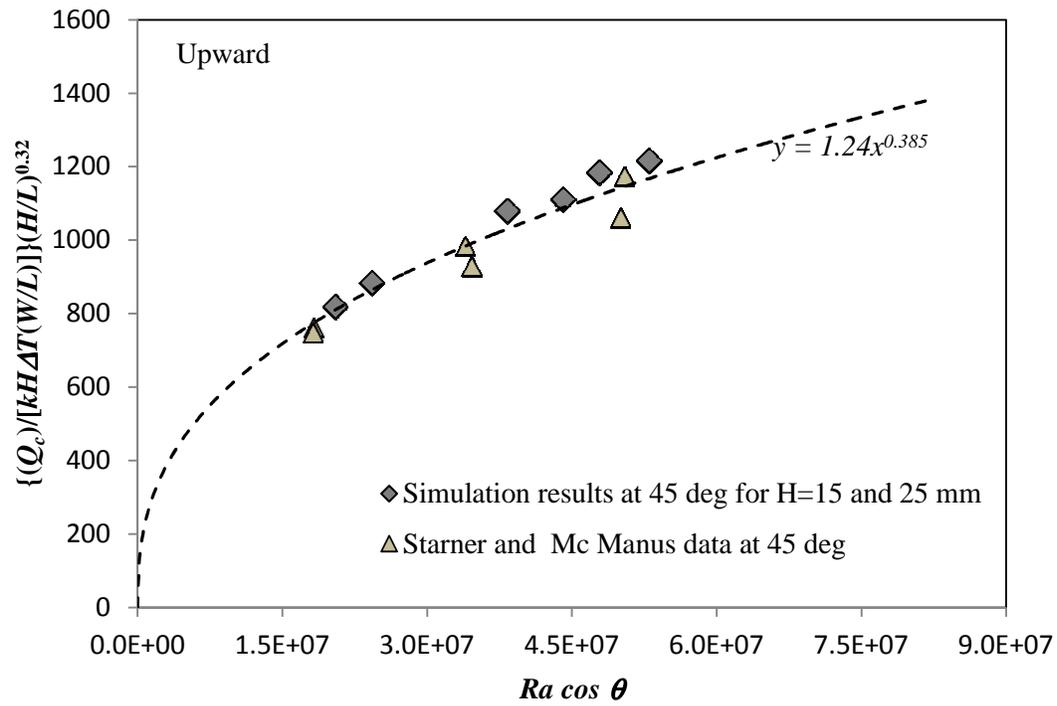


Fig. 12. Simulation results and Ref. [5] results at -45° plotted together with equation (4).

Highlights

- Natural convection heat transfer from inclined plate-fin heat sinks is investigated.
- A correlation for estimating convection heat transfer rates is suggested.
- The correlation is shown to be valid in a very wide range of angles, $-60^\circ \leq \theta \leq +80^\circ$.
- The correlation is verified with all available experimental data in literature.
- Flow separation and fin height play the most significant roles at high inclinations.